Non-Uniformly Sampled-Data Control of MIMO Systems

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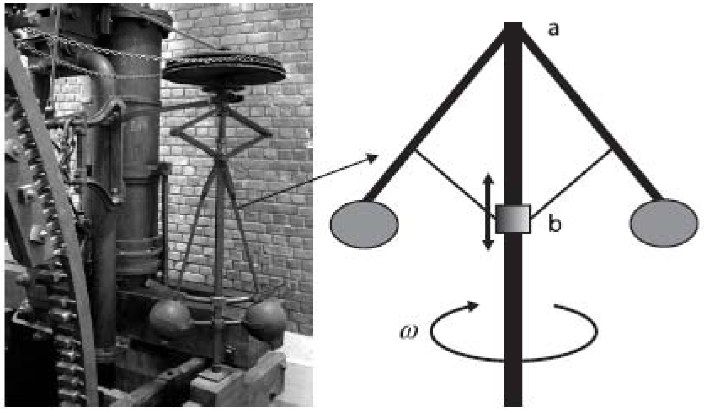
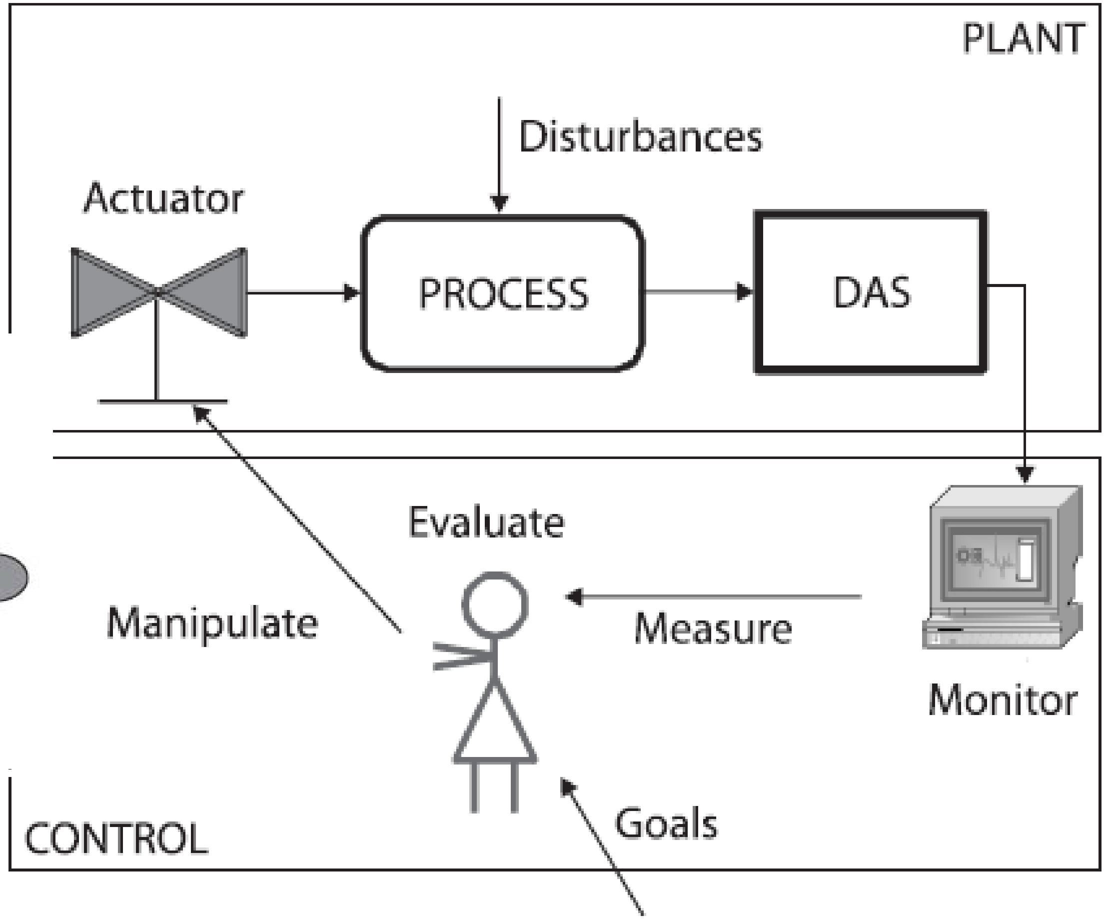
Control Paradigms

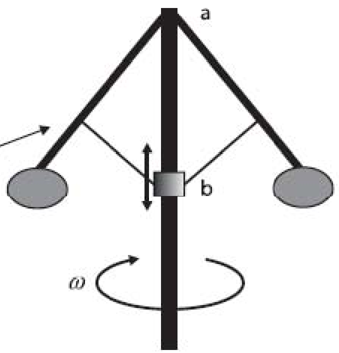
- Human Control

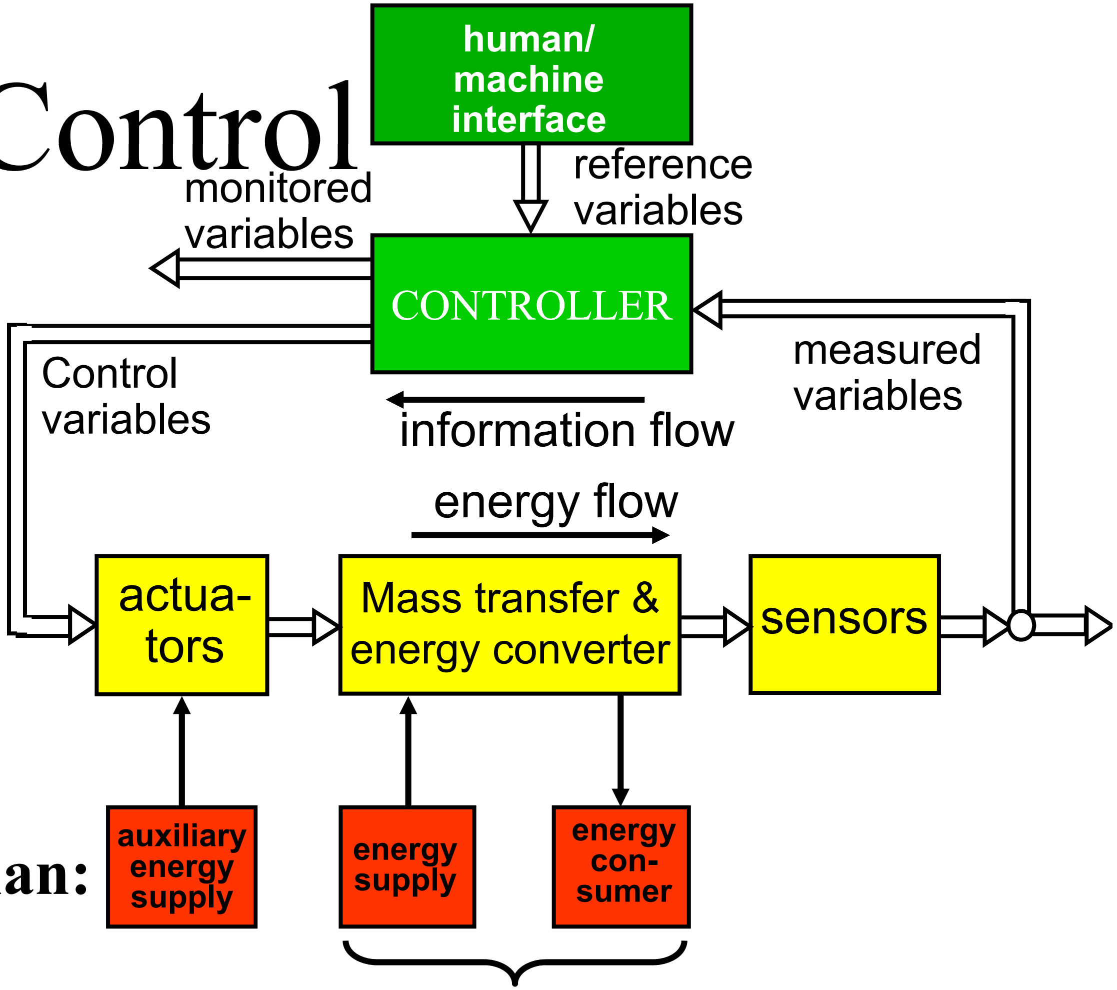




Control Paradigms

- Automatic Control





The controller is not Human:
It can do it better
(sometimes!)

Control Paradigms

- Digital Control

Information processing:

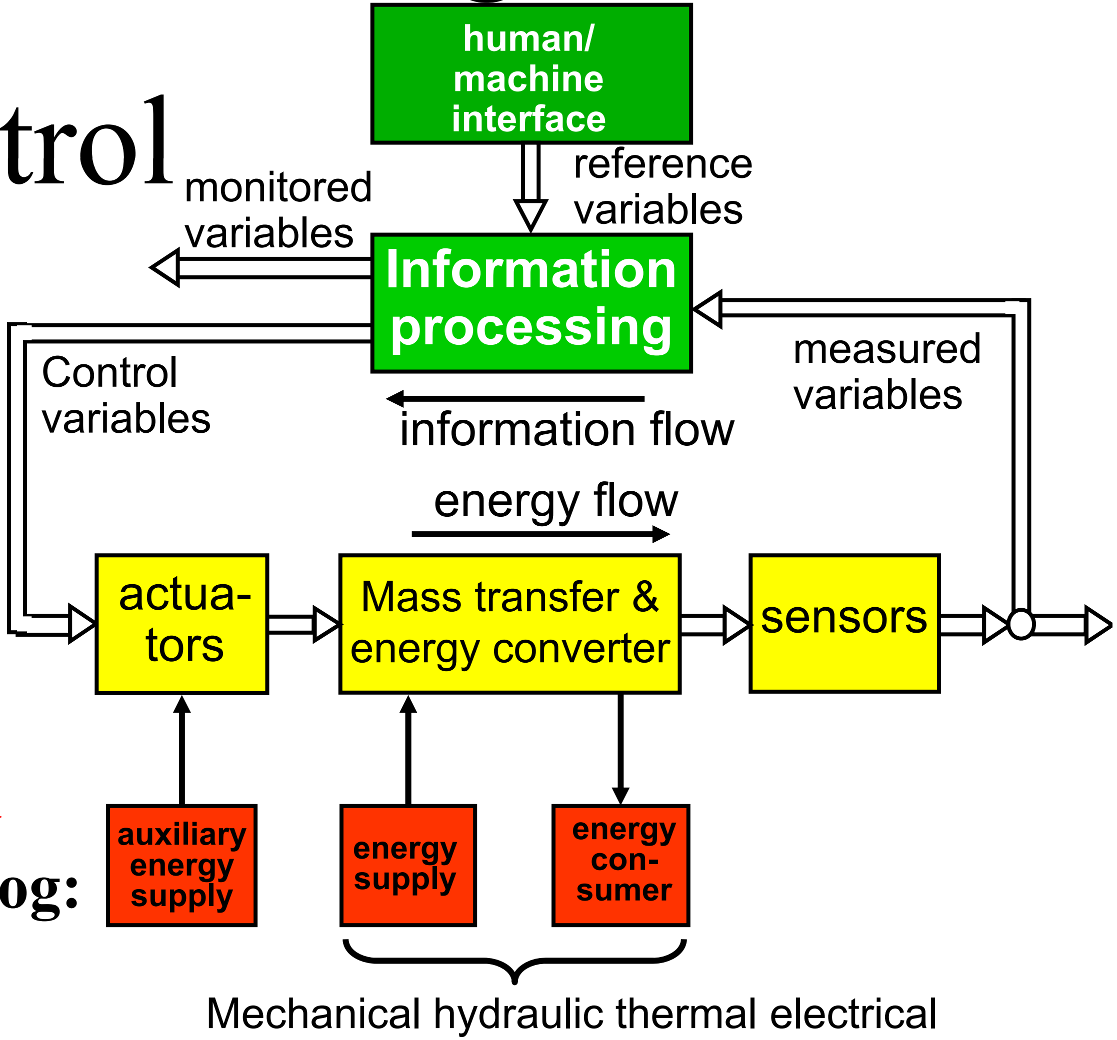
- Sequential operation
- Multitasking
- Resource constraints

- Control when needed

Digital control is not Analog:

It can do it better

(sometimes!)



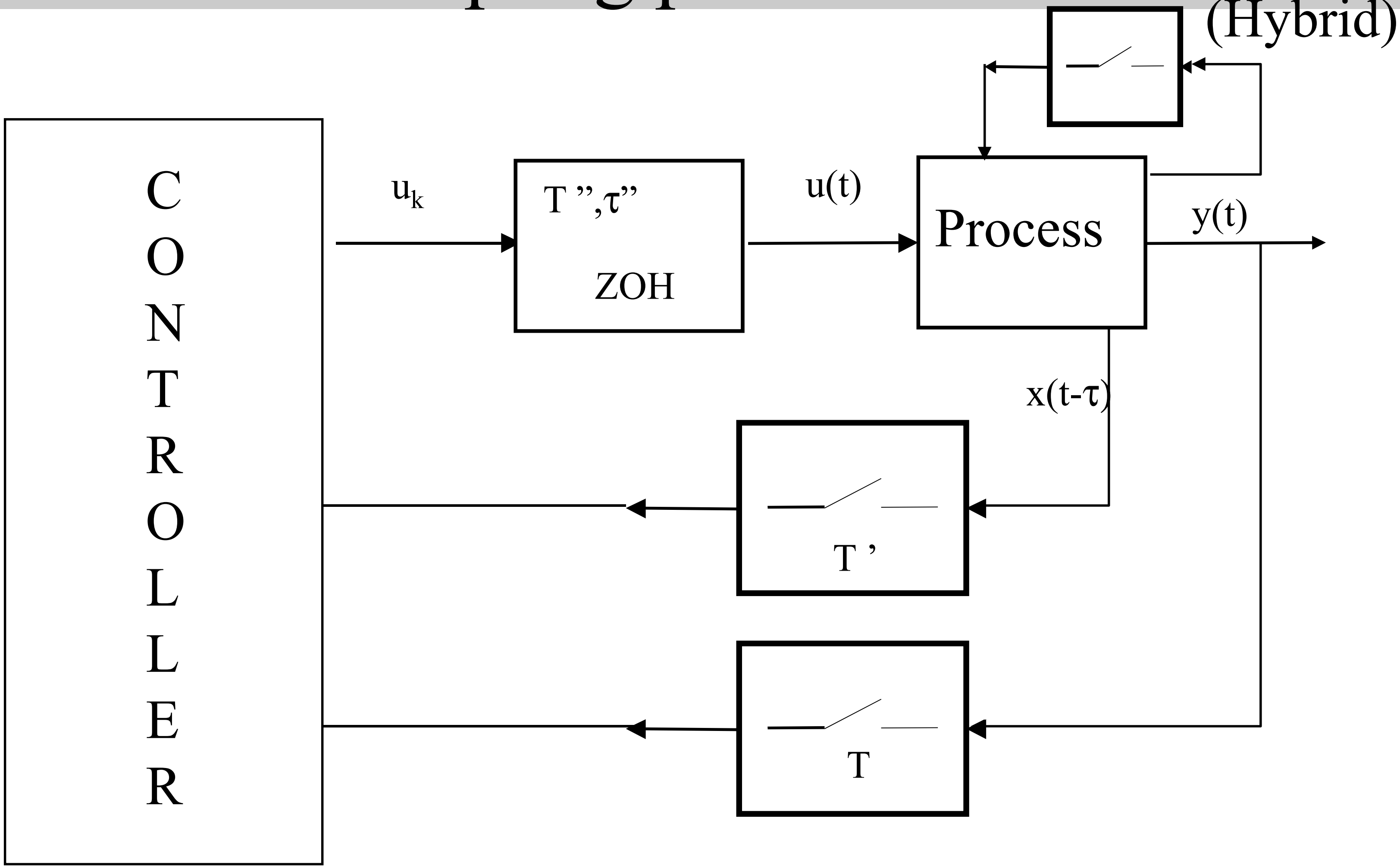
Control Paradigms

- Digital Control
  - Advantages
    - Cheap, flexible, versatile …
    - Portable, understandable …
    - New features …
  - Drawbacks
    - Open-loop
    - Interferences
    - Hidden oscillations

Outline

- Motivation for NUSD control
- Modeling NUSD systems
- Inference control
- S&H modeling and control
- Block Multirate control
- Ripple-free control
- Conclusions

Sampling pattern



Periods, T, as well as delays, τ, could be time variable

Motivation

- For MIMO systems:
  - Not all measurements are available at the same rate
  - Control actions can be updated faster
  - Bandwidth can be enlarged (faster response)
  - Some data can be missing
  - Computing resources can be reduced

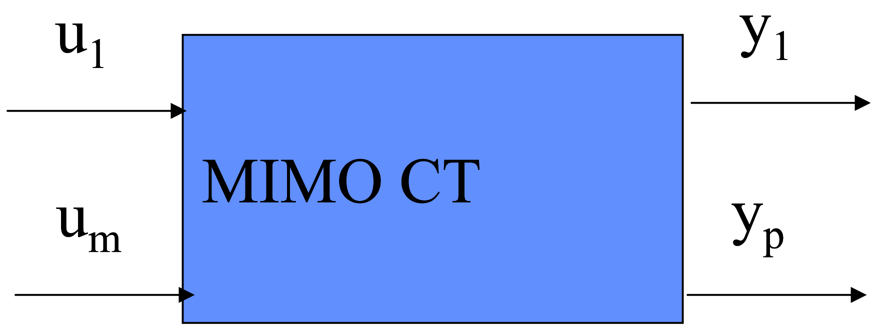
But:

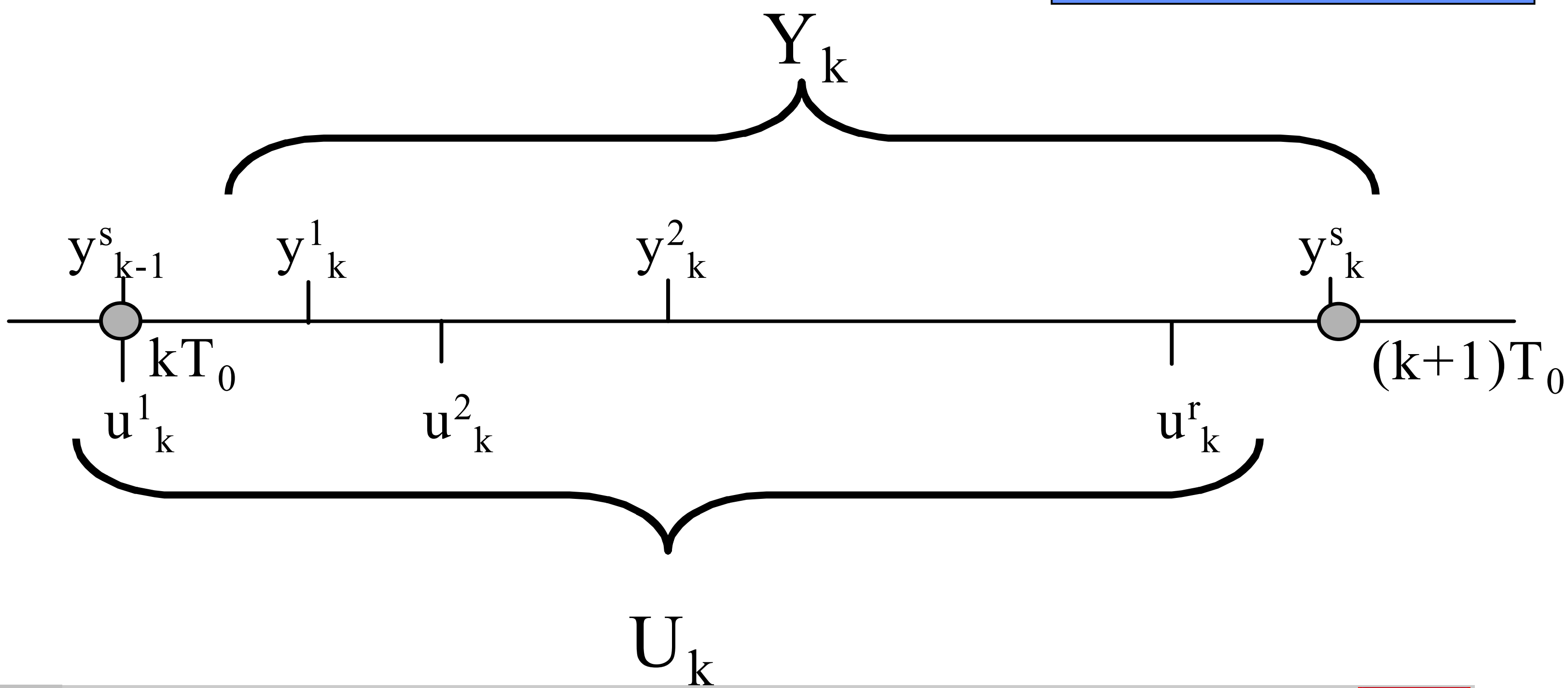
  - Controller design is not straightforward
  - Hidden oscillations may appear (ripple)

Motivation

Non uniform …

but periodic sampling pattern





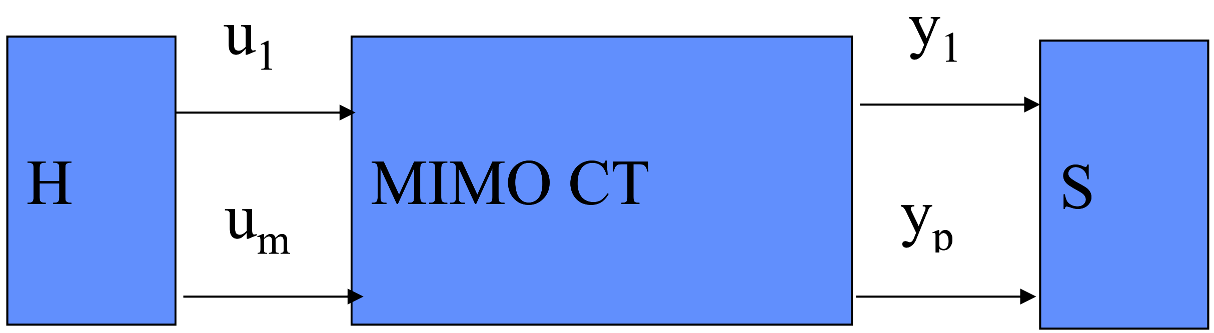
Historical Background

– Enlarged MIMO system,
  - Araki and Yamamoto, 1986
  - Berg, et al., 1988
  - Godbout, et al., 1990
  - Albertos, et al., 1996

→ LTI

- Vector Switch Decomposition
  - Kranc 1957
  - Thompson, 1986, 1988
- I/O Expansion
  - Meyer 1975
  - Albertos, 1990
- Lifting
  - Bamieh, 1990
  - Bamieh & Voulgaris, 1992

Historical Background (2)



- CT System + S&H Device
  - Kalman & Bertram, 1959
  - Tornero, 1983,1985
  - Longhi, 1994

Our Purpose today

Consider controlled processes where the sampling-updating pattern is ***NON-UNIFORM***.

Cases: Fast/slow signals
Distributed Control Systems
Networked Control Systems
Periodic Control

Approaches: Fast Inference Control
S&H Multirate Control
Block Multirate Input/output Control

Issues: Computational load
Hidden oscillations

Plant model

Continuous LTI System :

$$\dot{x}(t) = A_c x(t) + B_c u(t)$$
$$y(t) = Cx(t)$$
$$x \in R^n, u \in R^m, y \in R^p$$

B_c, C are full rank matrices

The system is controllable and observable.
μ is the controllability index, and
ν the observabiliy index

with input/output representation:

$$y(s) = G(s)u(s)$$

DT Plant model

Single rate fast and slow models:

Discrete LTI system, period T :

$$x_{k+1} = Ax_k + Bu_k$$

$$y_k = Cx_k$$

$$x \in R^n, u \in R^m, y \in R^p$$

$$A = e^{A_C T}$$

$$B_q = \int_0^{qT} e^{A_C \tau} d\tau . B_C ; B = B_1$$

$$y(z) = G(z)u(z)$$

Discrete LTI system, period $NT = T_o$:

$$x_{k+1} = A^N x_k + B_N u_k$$

$$y(z^N) = G_N(z^N)u(z^N)$$

Control design goal

- Control based on fast model could be more performing

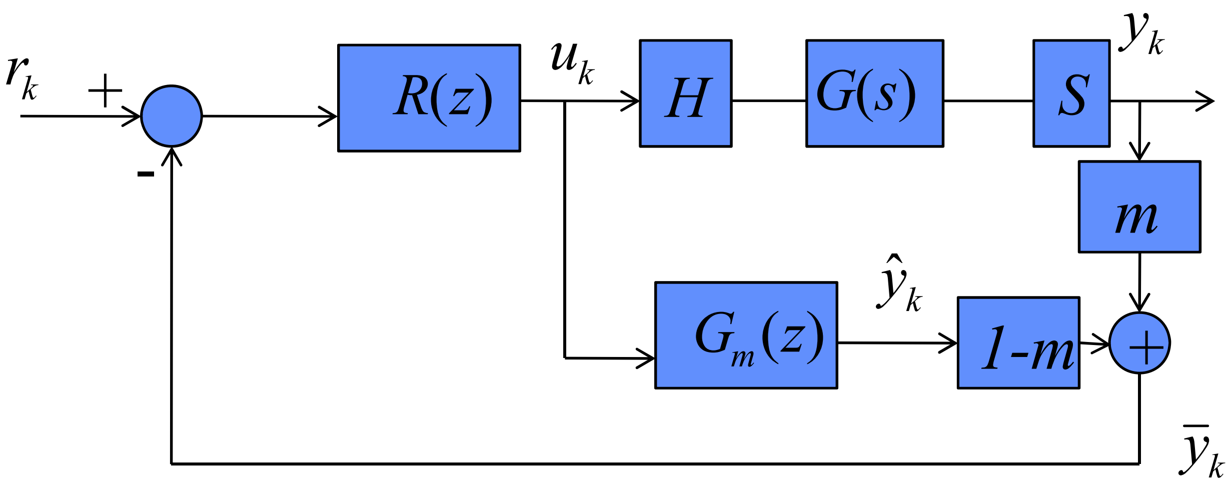
… but it requires more data and computing resources

- Control based on slow model can be improved if:
  - Additional information
  - Faster control updating

Goal: get the best performance with the available sampling/updating pattern.

Inference control

SISO fast input



where $m = \{1,0,0,\ldots,1,0,..\}$

Convergence issues:

- Uncertainty
- Disturbances
- Instability

Inferenced feedback control

Fast control based on inferenced output: mixed regressor

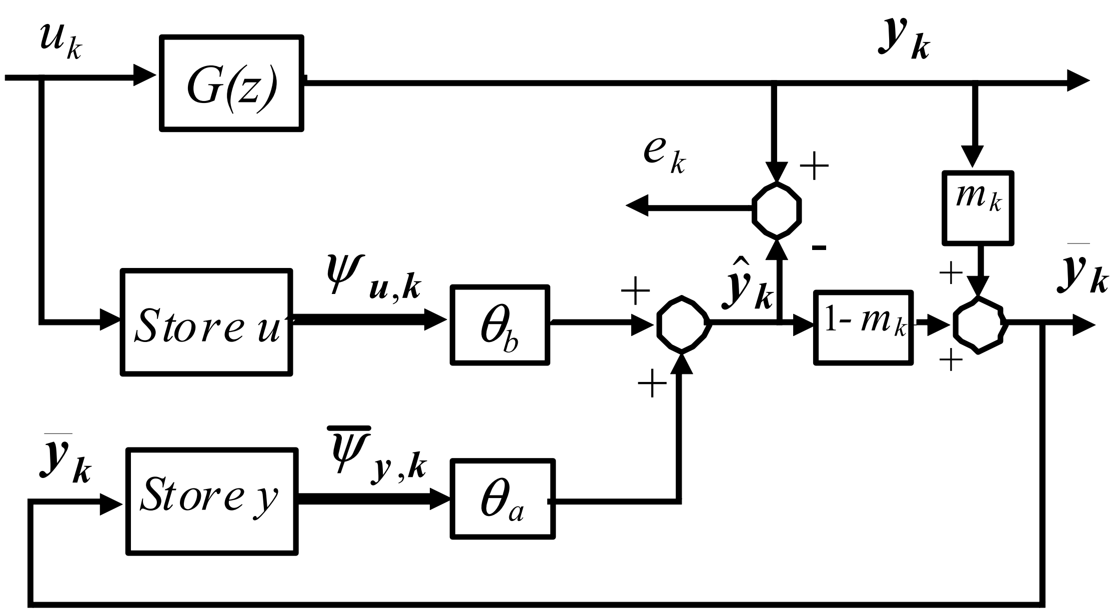
Data availability may be random. In [1] conditions on:

- sampling period
- scarcity

$$\hat{y}_k = -\sum_{i=1}^{n} a_i \hat{y}_{k-i} + \sum_{i=1}^{n} b_i u_{k-i} = \hat{\psi}_{k-1}{}^T \cdot \theta$$

$$\hat{y}_k = \theta_b \psi_{u,k} + \theta_a \hat{\psi}_{y,k}$$

$$\Rightarrow \hat{y}_k = \theta_b \psi_{u,k} + \theta_a \overline{\psi}_{y,k}$$

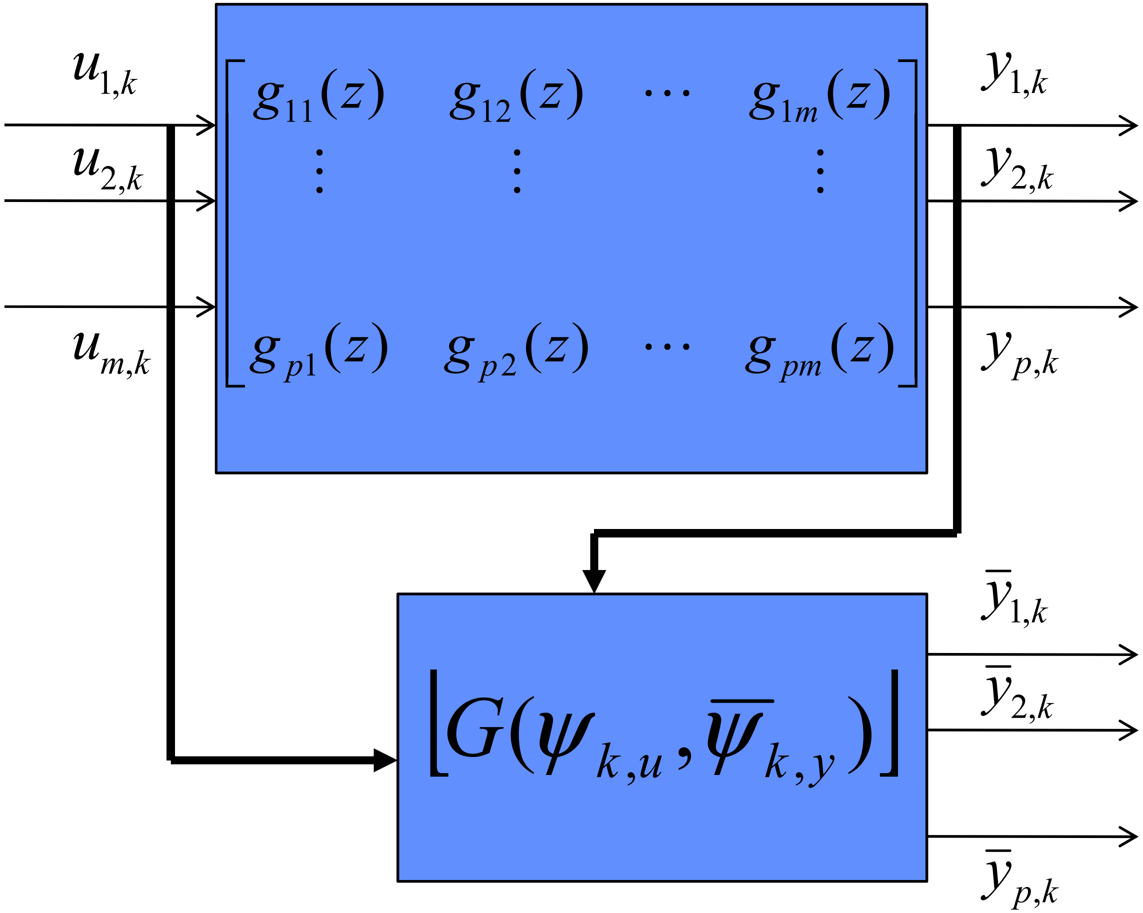


[1] OUTPUT PREDICTION UNDER SCARCE DATA OPERATION. CONTROL APPLICATIONS.
P. ALBERTOS, R. SANCHIS, A. SALA Automatica 35 (1999) 1671-1681

Mixed regressor predictor

- Advantages:
  - simplicity and easy operation
  - Random and missing data
- Drawbacks: Convergence depends on
  - Sampling period
  - Data availability
  - Process dynamics
- Alternatives:
  - **Periodic** Kalman filter

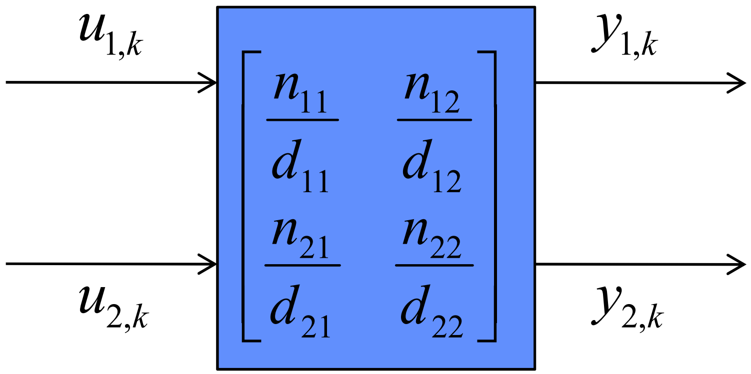
MIMO Inferenced feedback control



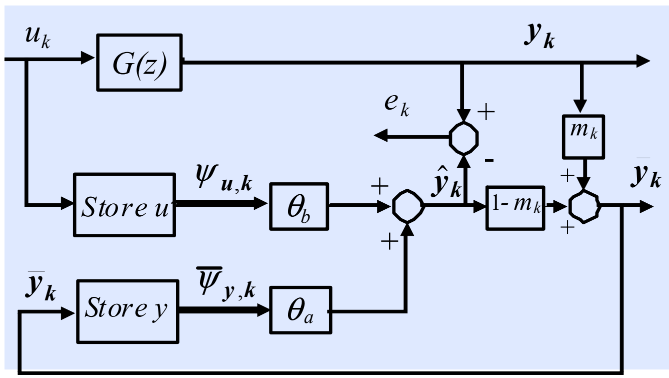
MIMO fast control: No ripple, comp. load, convergence constraints

MIMO Inferenced feedback control

TITO fast control:



$$d_{11}d_{12}y_1(z) = n_{11}d_{12}u_1(z) + n_{12}d_{11}u_2(z)$$
$$d_{21}d_{22}y_2(z) = n_{21}d_{22}u_1(z) + n_{22}d_{21}u_2(z)$$



$$\Rightarrow \begin{cases} \hat{y}_{1,k} = \theta_{b_{11}}\psi_{u1,k} + \theta_{b_{12}}\psi_{u2,k} + \theta_{a1}\bar{\psi}_{y1,k} \\ \hat{y}_{2,k} = \theta_{b_{21}}\psi_{u1,k} + \theta_{b_{22}}\psi_{u2,k} + \theta_{a2}\bar{\psi}_{y2,k} \end{cases}$$

MIMO fast control: No ripple, comp. load, convergence constraints

Academic example

$$G(z)=\begin{bmatrix}\frac{0.3066(z+0.4709)}{(z-0.3679)(z-0.2865)} & \frac{-0.37927}{z-0.3679}\\ \frac{0.30277(z+0.7835)}{(z-1.649)(z-0.2865)} & \frac{0.64872}{z-1.649}\end{bmatrix};T=0.5s$$

Fast update

Measurements:

$$m_1=\{1\quad 0\quad 0\quad 1\quad 0\quad 0\}$$
$$m_2=\{1\quad 0\quad 1\quad 0\quad 1\quad 0\}$$

$$\Rightarrow\begin{cases}\hat{y}_{1,k}=[0.3066\quad 0.1444]\psi_{u1,k}+[-0.37927\quad 0.1087]\psi_{u2,k}+[-0.6544\quad .1054]\overline{\psi}_{y1,k}\\ \hat{y}_{2,k}=[0.3028\quad 0.2372]\psi_{u1,k}+[0.64872\quad 0.1859]\psi_{u2,k}+[-1.935\quad 0.4724]\overline{\psi}_{y2,k}\end{cases}$$

$$\psi_{u1,k}=\begin{bmatrix}u_{1,k-1}\\u_{1,k-2}\end{bmatrix};\psi_{u2,k}=\begin{bmatrix}u_{2,k-1}\\u_{2,k-2}\end{bmatrix};\overline{\psi}_{y1,k}=\begin{bmatrix}\bar{y}_{1,k-1}\\\bar{y}_{1,k-2}\end{bmatrix};\overline{\psi}_{y2,k}=\begin{bmatrix}\bar{y}_{2,k-1}\\\bar{y}_{2,k-2}\end{bmatrix}$$

$$\bar{y}_{1,k-1}=m_{1,k-1}y_{1,k-1}+(1-m_{1,k-1})\hat{y}_{1,k-1}$$
$$\bar{y}_{2,k-1}=m_{2,k-1}y_{2,k-1}+(1-m_{2,k-1})\hat{y}_{2,k-1}$$

Reactor example

Unstable Batch Reactor, benchmark[1]

a =

1.3800	-0.2077	6.7150	-5.6760
-0.5814	-4.2900	0	0.6750
1.0670	4.2730	-6.6540	5.8930
0.0480	4.2730	1.3430	-2.1040

b =

0	0
5.6790	0
1.1360	-3.1460
1.1360	0

c =

1	0	1	-1
0	1	0	0

d = **0**

PI, decentralized control:

$$K_{12}(s) = -\frac{5s+8}{s}; K_{21}(s) = \frac{2s+2}{s}$$

Sampling period T=0.01s;

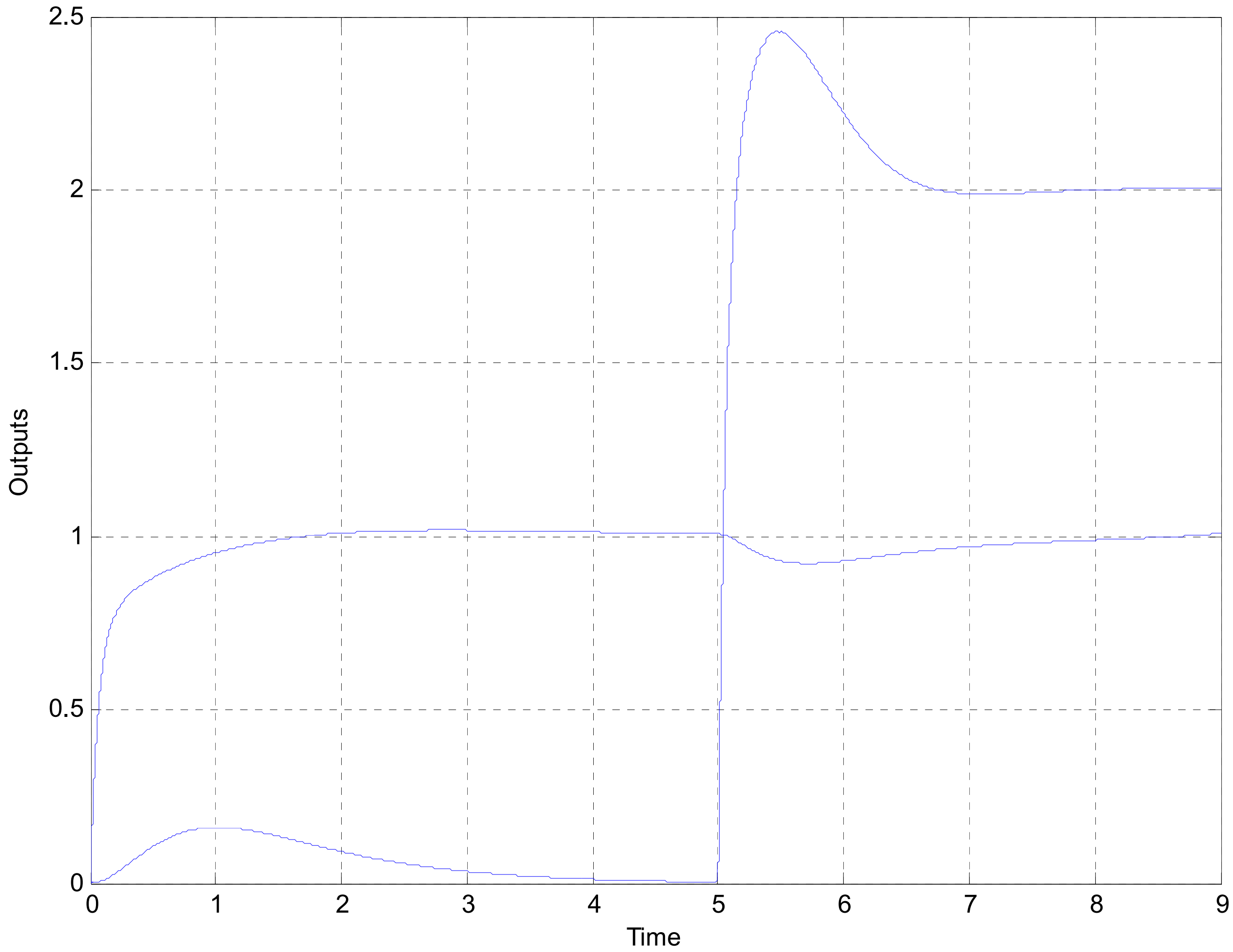
Missing data:

First output: [0 0.03 0.05], lost in 0.01, 0.02 and 0.04

Second output: [0 0.01 0.03 0.04], lost in 0.02 and 0.05.

[1] *Stability Analysis of Networked Control Systems*, Gregory C. Walsh, Hong Ye, and Linda G. Bushnell. IEEE Trans on Control Systems Technology, Vol. 10, no. 3, May 2002

Reactor responses



Multirate Hold model [2]

Continuous LTI System :

$$\dot{x}(t) = A_c x(t) + B_c u(t)$$
$$y(t) = Cx(t)$$

Discrete LTI system, period T :

$$x_{k+1} = Ax_k + Bu_k$$
$$y_k = Cx_k$$
$$A = e^{A_C T}$$
$$B_q = \int_0^{qT} e^{A_C \tau} d\tau . B_C ; B = B_1$$

Virtual hold Updating rate:

$$u_i \to N_i T$$

$$\Delta_k \equiv diag\{\delta_i(k), i = 1,2,\dots,m\}$$
$$\delta_i(k) \equiv \begin{cases} 1, & k = j \cdot N_i, \\ 0 & k \neq j \cdot N_i, \end{cases} \quad j \in Z^+$$

$$x_{k+1} = Ax_k + B(I - \Delta_k)v_k + B\Delta_k u_k$$
$$v_{k+1} = (I - \Delta_k)v_k + \Delta_k u_k$$

[2] PERIODIC OPTIMAL CONTROL OF MULTIRATE SAMPLED DATA SYSTEMS.
J. TORNERO, P. ALBERTOS, J. SALT PSYCO 2001. COMO (ITALY)

Multirate Sampled model

Continuous LTI System :

$$\dot{x}(t) = A_c x(t) + B_c u(t)$$
$$y(t) = Cx(t)$$

Discrete LTI system, period T :

$$x_{k+1} = Ax_k + Bu_k$$
$$y_k = Cx_k$$
$$A = e^{A_C T}$$
$$B_q = \int_0^{qT} e^{A_C \tau} d\tau . B_C ; B = B_1$$

Sampling rate: $y_i \rightarrow M_i T$

$$\overline{\Delta}_k \equiv diag\{\overline{\delta}_i(k), i = 1,2,\ldots,p\}$$
$$\overline{\delta}_i(k) \equiv \begin{cases} 1, & k = j \cdot M_i, \\ 0 & k \neq j \cdot M_i, \end{cases} \quad j \in Z^+$$

$$y_k = \overline{\Delta}_k C x_k = C_k x_k$$

Multirate S&H Model (2)

Altogether:

$$\bar{x}_k \equiv \begin{bmatrix} x_k \\ v_k \end{bmatrix} \in R^{n+m} \qquad \bar{x}_{k+1} = A_{M,k} \cdot \bar{x}_k + B_{M,k} \cdot u_k$$

$$A_{M,k} = \begin{bmatrix} A & B \cdot (I - \Delta_k) \\ 0 & I - \Delta_k \end{bmatrix} \qquad B_{M,k} = \begin{bmatrix} B \cdot \Delta_k \\ \Delta_k \end{bmatrix}$$

Hold

$$[\bar{x}]_{k+1} = [A_{M,k} \quad B_{M,k}] \cdot \begin{bmatrix} \bar{x} \\ u \end{bmatrix}_k = \Phi \cdot H \begin{bmatrix} \bar{x} \\ u \end{bmatrix}_k = \begin{bmatrix} A & B \\ 0 & I \end{bmatrix} \cdot \begin{bmatrix} I & 0 & 0 \\ 0 & I - \Delta_k & \Delta_k \end{bmatrix} \cdot \begin{bmatrix} x \\ v \\ u \end{bmatrix}_k$$

plant

Periodic LQG Control

- SR LQ control: $J = \frac{1}{2}\sum_{k=0}^{\infty}\left\{ x_k{}^T Q x_k + u_k{}^T R u_k \right\}$ $\quad u_k = K.x_k$
- MR LQ control:

Direct control — Held control

$$J = \frac{1}{2}\sum_{k=0}^{\infty}\{x_k{}^T \cdot Q \cdot x_k + u_k{}^T \Delta_k^T R \Delta_k \cdot u_k + v_k{}^T [I-\Delta_k]^T R [I-\Delta_k] \cdot v_k\}$$

$$J_M = \frac{1}{2}\sum_{0}^{\infty}\begin{bmatrix} x_k{}^T & v_k{}^T & u_k{}^T \end{bmatrix} \cdot \overline{Q}_M(k) \cdot \begin{bmatrix} x \\ v \\ u \end{bmatrix}_k$$

$$\overline{Q}_{M,k} = \begin{bmatrix} I & 0 & 0 \\ 0 & I-\Delta_k & \Delta_k \end{bmatrix}^T \begin{bmatrix} Q & 0 \\ 0 & R \end{bmatrix} \begin{bmatrix} I & 0 & 0 \\ 0 & I-\Delta_k & \Delta_k \end{bmatrix} = \begin{bmatrix} Q_{M,k} & M_{M,k} \\ M_{M,k} & R_{M,k} \end{bmatrix}$$

MR LQR Solution : Periodic Control K(k)

$$u_k = K_k \bar{x}_k - R_{M,k}^{-1} M_{M,k} \bar{x}_k$$

$$\bar{x}_k = \begin{bmatrix} x \\ \nu \end{bmatrix}_k$$

$$K_k = \left[R_{M,k} + B_{M,k}^T P_{k+1} B_k\right]^{-1} B_{M,k}^T P_{k+1} \bar{A}_{M,k}$$

$$P_k = \bar{A}_{M,k}^T P_{k+1} \bar{A}_{M,k} + \bar{Q}_{M,k} - \bar{A}_{M,k}^T P_{k+1} B_{M,k} K_k$$

$$\bar{A}_{M,k} = A_{M,k} - B_{M,k} R_{M,k}^{-1} M_{M,k}$$

$$\bar{Q}_{M,k} = Q_{M,k} - M_{M,k} R_{M,k}^{-1} M_{M,k}$$

Periodic State Estimator

- Given the system

$$x_{k+1} = Ax_k + Bu_k + w_k$$
$$y_k = C_k x_k + \eta_k$$
$$C_k = \overline{\Delta}_k C$$

$$\overline{\Delta}_k \equiv diag\{\overline{\delta}_i(k), i = 1,2,\ldots,p\}$$

$$\overline{\delta}_i(k) \equiv \begin{cases} 1, & k = j \cdot M_i, \\ 0 & k \neq j \cdot M_i, \end{cases} \quad j \in Z^+$$

define the periodic state estimator

$$\hat{x}_{k+1} = Ax_k + Bu_k + L_k[y_k - C_k \hat{x}_k]$$

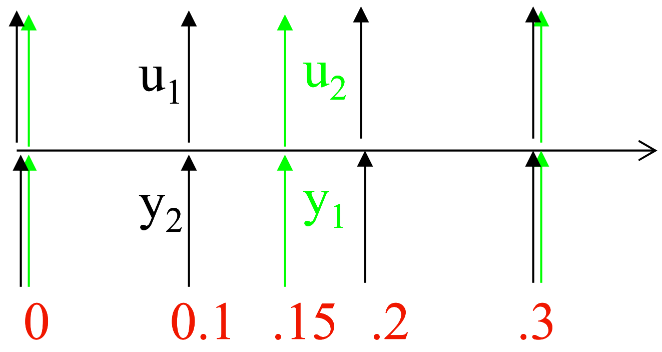
$$L_k = AP_{k+1}C_k^T \left[R_2 + C_k P_k C_k^T\right]^{-1}$$
$$P_{k+1} = AP_k A^T + R_1 - L_k C_k P_k A^T$$

Illustrative Example

- Assume the CT system

$$A_c = \begin{bmatrix} -2.5 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 1 \end{bmatrix}; B_c = \begin{bmatrix} 2.5 & 0 \\ 10 & -1.2 \\ 5/6 & 1 \end{bmatrix}; C = \begin{bmatrix} -4 & 1 & 0 \\ -1/3 & 0 & 1 \end{bmatrix}$$

with sampling/updating pattern:



T=0.5

$$u_k = K_i \bar{x}_k ; i = k \bmod (N)$$

Tim
K(
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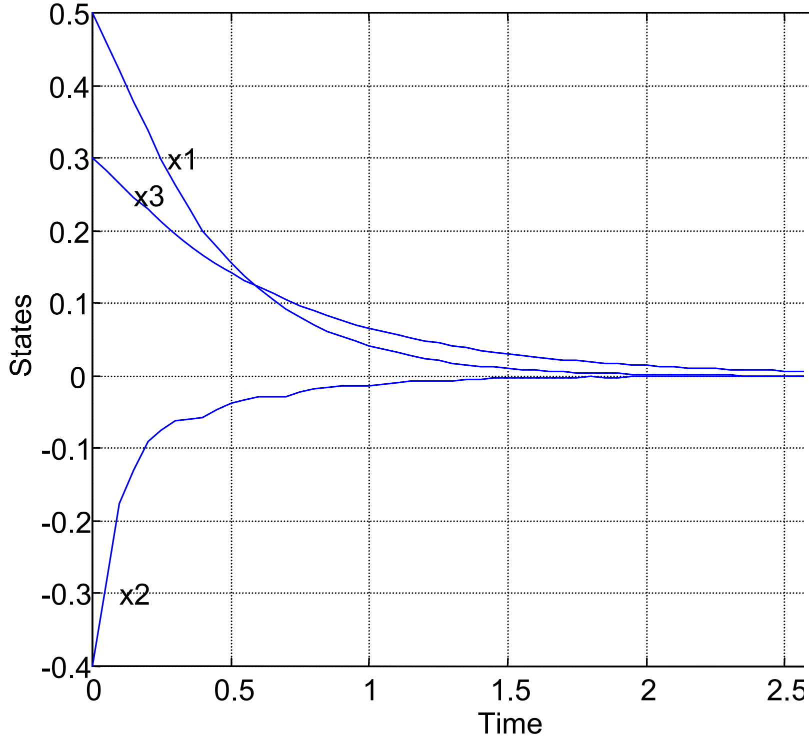
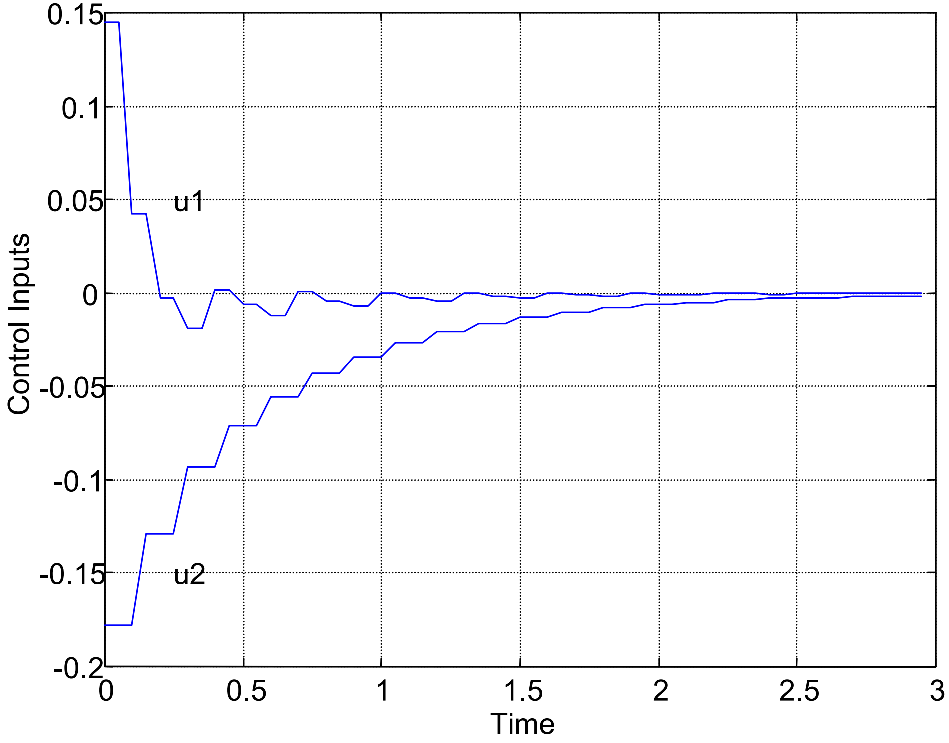
Time	Filter Gain
L(0)	$\begin{matrix} -0.2002 & -0.0125 \\ 0.0614 & -0.0360 \\ -0.0558 & 0.6781 \end{matrix}$
L(1)	**0**
L(2)	$\begin{matrix} 0 & -0.1413 \\ 0 & 0.0034 \\ 0 & 0.6426 \end{matrix}$
L(3)	$\begin{matrix} -0.2006 & 0 \\ 0.0601 & 0 \\ -0.0317 & 0 \end{matrix}$
L(4)	$\begin{matrix} 0 & -0.0860 \\ 0 & -0.0139 \\ 0 & 0.6590 \end{matrix}$
L(5)	**0**

Time responses

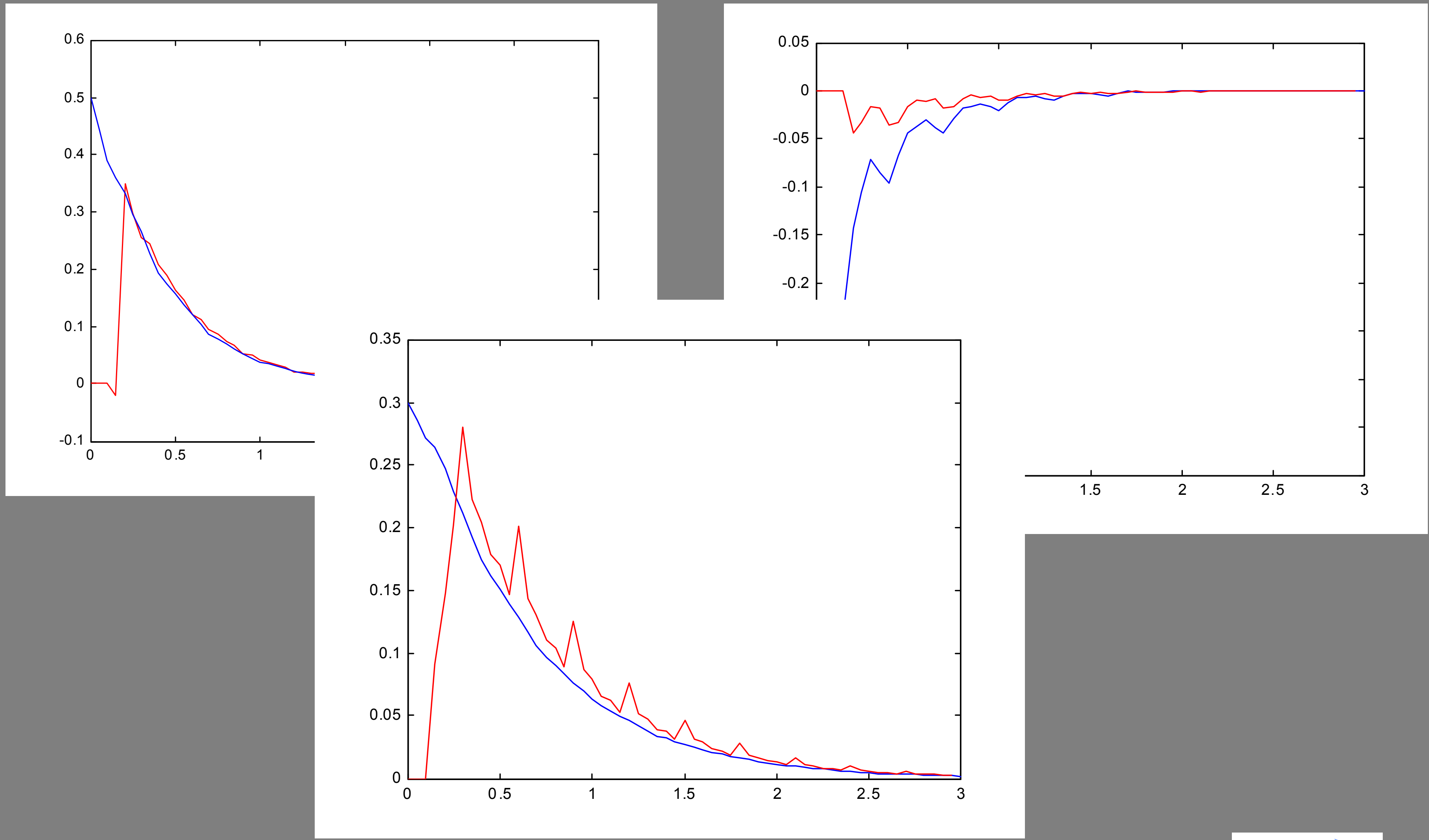
$$x^T(0) = [0.5 \quad -0.4 \quad 0.3]$$





Non-Uniformly

Observer effect



S&H Multirate approach

- Compact model with LTI + S&H periodic device
  - Easy to change the S and/or Hold strategy
- Multirate LQR easily solved as set of Riccati equation
- Multirate LQG easily solved using a set of Kalman filter equations
- Three blocks in the system:
  - the plant (LTI, low order)
  - the periodic controller and
  - the periodic estimator
- Clear understanding
- Oscillatory effects. Bang-bang

BMIO Approach [3]

- Consider blocks of inputs and outputs
- Slow state representation
- Arbitrary but periodic S&H pattern
- Assign the slow plant structure
- Track arbitrary reference
- Hidden oscillations
- Intersampling behavior

[3] BLOCK MULTIRATE INPUT-OUTPUT MODEL FOR SAMPLED DATA CONTROL SYSTEMS. P. ALBERTOS, IEEE Trans on AC. Vol 35 N9 1085-1088. 1990.

BMIO Procedure

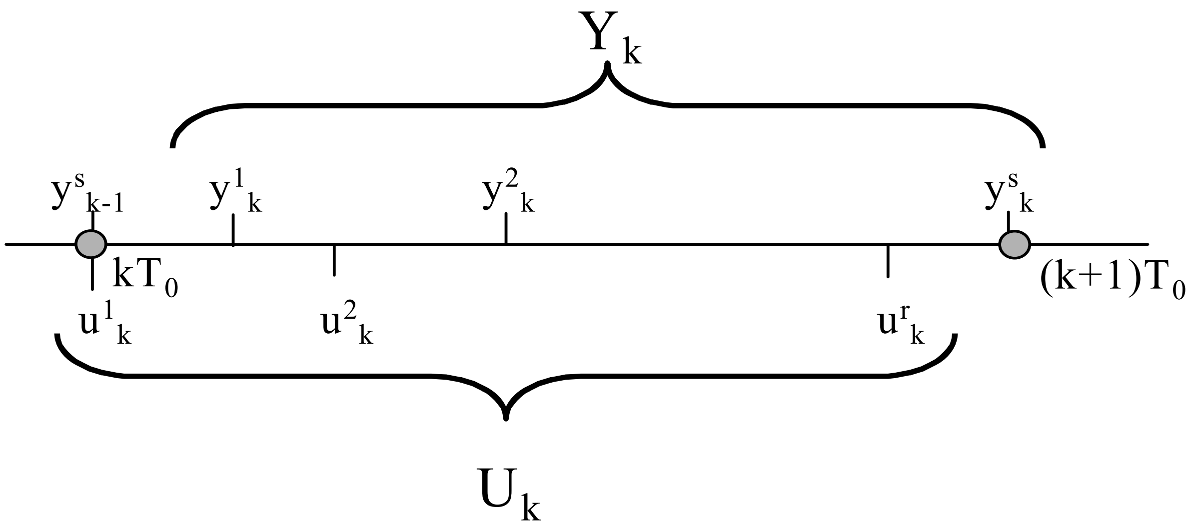
Discrete LTI system, period T :

$$x_{k+1} = Ax_k + Bu_k$$

$$y_k = Cx_k$$

$$A = e^{A_C T}$$

$$B_q = \int_0^{qT} e^{A_C \tau} d\tau . B_C ; B = B_1$$



Periodic system with (evolving) period T_o and
- Enough number of measurements (> ν)
- Enough number of actions (> μ)

$s_i = t_i / T$, $r^j = t^j / T$ and N= T_o / T

$y(kT_0 + t_i), i = 1,2,\ldots,s$

$u(kT_0 + t^j), j = 1,2,\ldots,r$

Ouputs/Inputs defined in a frame period

BMIO Procedure.

Blocks Definition

$$Y_k = \begin{bmatrix} y(kT_0 + t_1) \\ y(kT_0 + t_2) \\ \ldots \\ y(kT_0 + t_s) \end{bmatrix}; \quad U_k = \begin{bmatrix} u(kT_0 + t^1) \\ u(kT_0 + t^2) \\ \ldots \\ u(kT_0 + t^r) \end{bmatrix}$$

N-period discrete ss representation

$$x_{k+1} = x[(k+1)T_0] = A^N x_k + WU_k$$

Block Controllability Matrix

$$W = \begin{bmatrix} A^{N-r_2} B_{r_2 - r_1} & \ldots & A^{N-r_r} B_{r_r - r_{r-1}} & B_{N-r_r} \end{bmatrix} \in R^{nxmr}$$

BMIO Procedure

$$Y_k = Ox_k + HU_k \qquad O = \begin{bmatrix} CA^{s_1} \\ CA^{s_2} \\ \\ CA^{s_s} \end{bmatrix} \in R^{ps \times n}$$

Block Observability Matrix

$$H = \begin{bmatrix} h_{11} & \dots & 0 \\ \vdots & \vdots & \vdots \\ h_{s1} & \dots & h_{sr} \end{bmatrix} \in R^{ps \times mr}$$

Haenkel Input-Output Matrix.

$$h_{ij} = CA^I B_J; \quad \begin{cases} r_j < s_i & I = s_i - r_j & J = r_j - r_{j-1} \\ r_{j-1} < s_i < r_j & I = 0 & J = s_i - r_{j-1} \\ s_i < r_{j-1} & I = 0 & J = 0 \end{cases}$$

Internal Representation of a Non-Uniformly SD system

$$x_{k+1} = x[(k+1)T_0] = A^N x_k + WU_k \qquad Y_k = Ox_k + HU_k$$

Remark: The matrices W, O, and H could be obtained by other modeling procedure.

If rank(O)=n, by state reconstruction.

# Denotes pseudoinverse

$$x_k = A^N x_{k-1} + WU_{k-1} = A^N O^{\#} Y_{k-1} + (W - A^N O^{\#} H) U_{k-1}$$

$$x_k = PY_{k-1} + QU_{k-1}$$

$$Y_k = OPY_{k-1} + OQU_{k-1} + HU_k$$

BMIO Procedure. Remarks

- If there are not enough samples to make O full rank matrix, the frame period must be enlarged (replicated)
- The block vectors could be shifted for a frame period starting at any other control updating time ⇒

⇒ W, O, H will be correspondingly changed.

PROPOSITION 1: For a frame period T_0

$$rank[W] = rank\lfloor b_1 \quad \dots \quad A_c^{g_1-1} b_1 \quad \dots \quad b_m \quad \dots \quad A_c^{g_m-1} b_m \rfloor$$

b_i is the i-column of the input matrix B_c

g_i updates for each input vector component u_i

The expression is equivalent to the previous one

$$W = \lfloor A^{N-r_2} B_{r_2-r_1} \quad \dots \quad A^{N-r_r} B_{r_r-r_{r-1}} \quad B_{N-r_r} \rfloor \in R^{nxmr}$$

Lemma: W is full rank, if $r \geq \mu \Leftrightarrow g_i \geq \mu_i$

Remarks: i) Dual results for observability.

ii) If W,O are full rank, they are non-singular in a SISO case

Analysis of the BMIO procedure.

PROPOSITION 2: Given a SISO non-uniformly SD system, the necessary conditions for rank(H)=s are:

1. $r \geq s$
2. $i = 1,\ldots,s \quad t^i \leq t_i$

Comments:

1. It is evident, based on the dimensions of H (sxr).

2. It points out the requirement of enough changes in the input components to ensure the desired output value at any sampling time.

3. The extension to MIMO system is not easy to express. (Firstly the system should be output controllable)

Analysis of the BMIO procedure.

PROPOSITION 3: For a MIMO non-uniformly SD system, the necessary conditions for rank(H)=ps are:

1. $mr \geq ps$
2. $i = 1,\ldots,s \qquad \sum_j m^j \leq p \qquad \forall j, \qquad t^j \leq t_i$

m^j is the number of inputs updated at time t^j

The sufficiency would require to check that any input component influences any output component between any two updating times.

✐ If H is full rank, the system will be denoted "*output invertible*".

✐ If condition 1 holds, it is always possible to start the frame period in such a way that condition 2 is also satisfied.

BMIO Model Controllability.

$$\left.\begin{array}{lcr} Y_{k+1} & = & OPY_k + OQU_k + HU_{k+1} \\ U_{k+1} & = & U_{k+1} \end{array}\right\}$$

$$\rightarrow \begin{bmatrix} Y_{k+1} \\ U_{k+1} \end{bmatrix} = \begin{bmatrix} OP & OQ \\ 0 & 0 \end{bmatrix} \begin{bmatrix} Y_k \\ U_k \end{bmatrix} + \begin{bmatrix} H \\ I \end{bmatrix} U_{k+1}$$

An enlarged "ss" representation. State dimension (ps+mr)

dim(O)=ps x n
dim(W)=n x mr

The controllability matrix is:

$$\begin{bmatrix} H & OPH + OQ \\ I & 0 \end{bmatrix} = \begin{bmatrix} H & OW \\ I & 0 \end{bmatrix}$$

mr x mr

In case of full rank matrices O, W the maximum rank of this matrix is mr+n

BMIO Model Controllability. Options

Using a new block vector defined as follows:

$$X_k = Y_k - HU_k = Ox_k$$

The new "state" equation is:

$$\begin{bmatrix} X_{k+1} \\ U_{k+1} \end{bmatrix} = \begin{bmatrix} OP & OPH - HOW \\ 0 & 0 \end{bmatrix} \begin{bmatrix} X_k \\ U_k \end{bmatrix} + \begin{bmatrix} 0 \\ I \end{bmatrix} U_{k+1}$$

✓ Only n eigenvalues of those attached to the dynamic of this enlarged state can be assigned by this control.

$$x_{k+1} = x[(k+1)T_0] = A^N x_k + WU_k \qquad Y_k = Ox_k + HU_k$$

$$x_k = PY_{k-1} + QU_{k-1} \qquad Y_k = OPY_{k-1} + OQU_{k-1} + HU_k$$

Structure Assignment Controller:

If W is full rank, a desired closed loop response $x_{k+1} = A_d x_k$ could be obtained applying the control law:

$$A_d x_k = A^N x_k + WU_k \Rightarrow U_k = W^{\#}(A_d - A^N)x_k = \Gamma x_k$$

State

How to choose it ?

Input / output

$$U_k = \Gamma PY_{k-1} + \Gamma QU_{k-1}$$

Output Tracking Controller:

If the block matrices W, O, and H are full rank, it is possible to obtain a desired reference.

$$Y_k = OPY_{k-1} + OQU_{k-1} + HU_k$$

Desired Reference

$$U_k = MY_{d,k} + M_1 Y_{k-1} + M_2 U_{k-1}$$

Input / Output Controller

$$M = H^{\#}; \quad M_1 = -H^{\#}OP; \quad M_2 = -H^{\#}OQ$$

and with reference forwarding:

StateFeedback Controller

$$U_k = H^{\#}(Y_{d,k} - Ox_k)$$

By comparison of:

$$U_k = W^{\#}(A_d - A^N)x_k = \Gamma x_k$$
$$U_k = H^{\#}(Y_{d,k} - O x_k)$$

$$W^{\#}(A_d - A^N) = -H^{\#}O$$

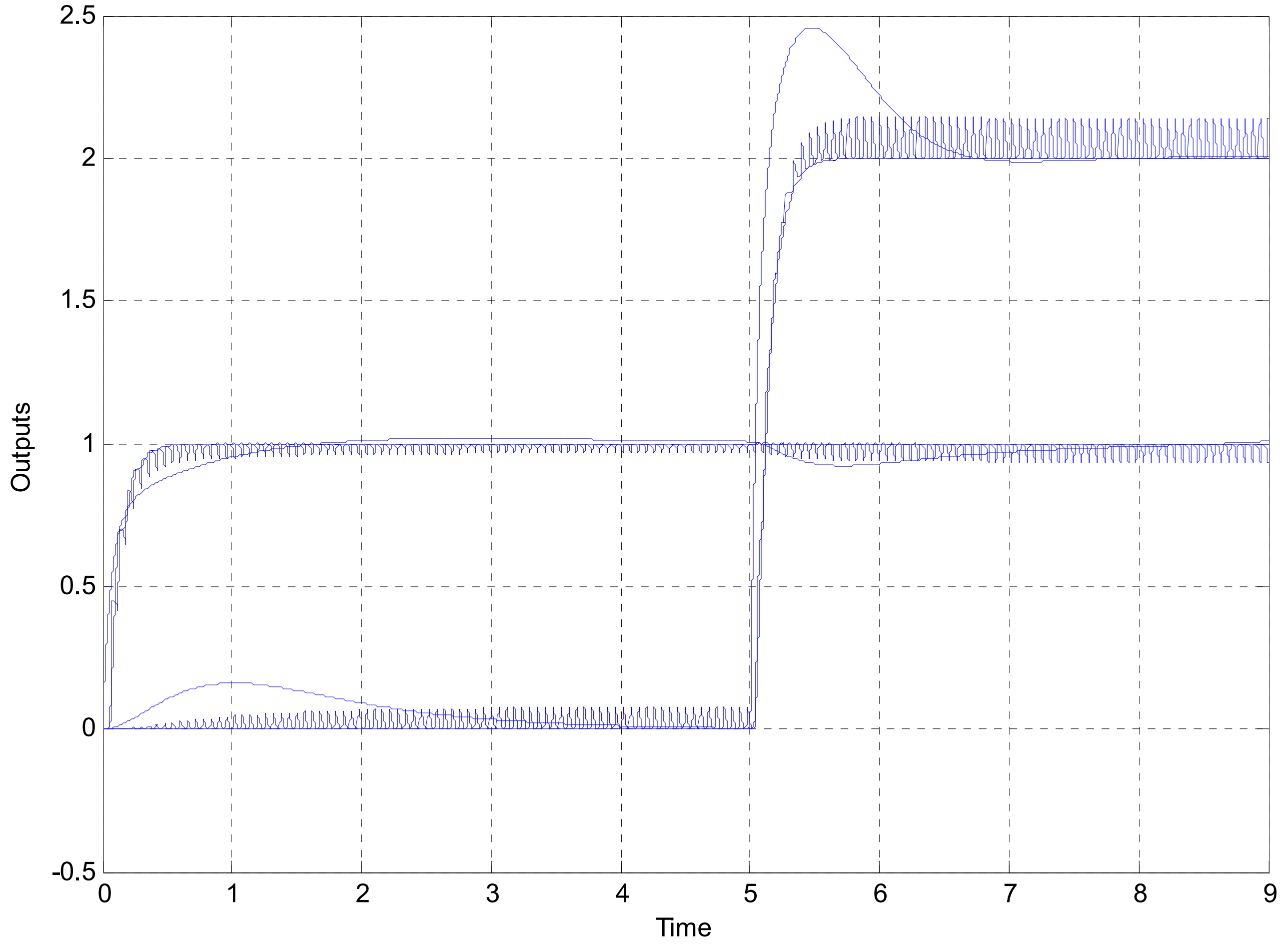
A perfect tracking of a given reference is equivalent to assigning a system structure $A_d = A^N - WH^{\#}O$

Previous controllers: Performance based in cancellation features.

Drawbacks: robustness, numerical computation accuracy, intersampling behaviour, disturbance treatment.

Reactor's example

BMIO reference tracking (compared with mixed regressor)



Another option: To force a suitable pole assignment.

If H is full rank (by proper selection of starting updating control):

$$U_k = MY_{d,k} + M_1 Y_{k-1} + M_2 U_{k-1}$$

This controller allows to assign mr+n poles of the global closed-loop controlled system by proper selection of matrices M's.

$$Y_k = OPY_{k-1} + OQU_{k-1} + HU_k$$

$$\begin{aligned} Y_k &= (OP + HM_1)Y_{k-1} + (OQ + HM_2)U_{k-1} \\ U_k &= M_1 Y_{k-1} + M_2 U_{k-1} \end{aligned}$$

The ouput reference has been deleted

The closed loop dynamic is characterized by the eigenvalues of:

$$\begin{bmatrix} OP + HM_1 & OQ + HM_2 \\ M_1 & M_2 \end{bmatrix} = \begin{bmatrix} OP & OQ \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} H \\ I \end{bmatrix} \underbrace{\begin{bmatrix} M_1 & M_2 \end{bmatrix}}$$

Data

Data

$M_1, \quad M_2$ are computed by any pole placement algorithm

Ripple-free Control.

In order to avoid the intersampling ripple of the controlled system, for constant inputs, a constant steady-state control should be used:

$$\bar{\Gamma} = \begin{bmatrix} K \\ \vdots \\ K \end{bmatrix}$$

Stability: $eig(A^N + W\Gamma)$

$$eig(A^N + \lfloor A^{N-r_2} B_{r_2 - r_1} + \ldots + A^{N-r_r} B_{r_r - r_{r-1}} + B_{N-r_r} \rfloor K)$$

Pole asignment: $(A^N, \lfloor A^{N-r_2} B_{r_2 - r_1} + \ldots + A^{N-r_r} B_{r_r - r_{r-1}} + B_{N-r_r} \rfloor)$

Regulation:

$$\Gamma_0 = W^{\#}(A_d - A^N)$$

$$U_k = \Gamma_k x_k; \qquad \Gamma_k \xrightarrow{\beta \to tuning} \begin{cases} \xrightarrow{initial} \Gamma_0 \\ \xrightarrow{final} \bar{\Gamma} \end{cases}$$

Ripple-free Control: Regulation

An dynamic control law can be expressed in matrix form

Initial control: $U_0 = \Gamma_0 x_0 = W^{\#}(A_d - A^N)x_0$

Steady-state control: $\overline{U} = \overline{\Gamma}\overline{x} = \begin{bmatrix} K \\ \vdots \\ K \end{bmatrix}\overline{x}$

Transient:

$$U_k = \Gamma_k x_k \qquad \Gamma_k = \beta\Gamma_{k-1} + (1-\beta)\overline{\Gamma}$$

The gain matrix should be reset for step changes in reference or disturbances

Ripple-free Control.

Tracking: $Y_{d,k}$

$$U_k = H^{\#}(Y_{d,k} - Ox_k) \qquad x_{k+1} = (A^N - WH^{\#}O)x_k + H^{\#}Y_{d,k}$$

Stability:

$$\Gamma_0 = -H^{\#}O \quad eig(A^N - WH^{\#}O) \quad eig(A^N + \left[A^{N-r_2}B_{r_2-r_1} + \ldots + A^{N-r_r}B_{r_r-r_{r-1}} + B_{N-r_r}\right]K)$$

$$\overline{\Gamma} = \begin{bmatrix} K \\ \vdots \\ K \end{bmatrix} \qquad \overline{\Gamma} = -\overline{H}^{\#}O \Rightarrow \begin{bmatrix} K \\ \vdots \\ K \end{bmatrix} \qquad \overline{M} = \overline{H}^{\#} = -\begin{bmatrix} K \\ \vdots \\ K \end{bmatrix} O^{\#} = \left[\overline{h}_{ij}\right]$$

Input/Output control:

$$U_k = MY_{d,k} + M_1 Y_{k-1} + M_2 U_{k-1}$$

$$M_0 = M = H^{\#}; \quad M_1 = -H^{\#}OP; \quad M_2 = -H^{\#}OQ$$

Ripple-free Control.

$$\Gamma(z) = \Gamma_0 + \frac{1-\beta}{1-\beta z^{-1}}(\overline{\Gamma} - \Gamma_0)$$

Input/Output control: $Y_{d,k}$

$$U_k = MY_{d,k} + M_1 Y_{k-1} + M_2 U_{k-1}$$

$$M = H_c^{\#}; \quad M_1 = -H_c^{\#} OP; \quad M_2 = -H_c^{\#} OQ$$

$$M_0 = H^{\#}; \qquad \overline{M} = \overline{H}^{\#} = -\begin{bmatrix} K \\ \vdots \\ K \end{bmatrix} O^{\#}$$

$$M(z) = M_0 + \frac{1-\beta}{1-\beta z^{-1}}(\overline{M} - M_0)$$

$$M_1(z) = -M(z)OP$$

$$M_2(z) = -M(z)OQ$$

$$M_k = \beta M_{k-1} + (1-\beta)\overline{M}$$

$$M_{1,k} = -M_k OP$$

$$M_{2,k} = -M_k OQ$$

Lag filter (scalar)

In order to avoid the intersampling ripple of the controlled system, for constant inputs, a constant steady-state control should be used.

The initial gain being γ_0, and the steady state gain being $\bar{\gamma}$, the controller will be: $u(z) = \gamma(z)x(z)$ where

$$\gamma(z) = \bar{\gamma}\frac{1-\alpha z^{-1}}{1-\beta z^{-1}} \Rightarrow \begin{cases} \beta \rightarrow tuning \\ \gamma(z) \xrightarrow[z=1]{} \gamma_0 \\ \gamma(z) \xrightarrow[z\rightarrow\infty]{} \bar{\gamma} \end{cases}$$

$$\gamma(1) = \bar{\gamma}\frac{1-\alpha}{1-\beta} = \gamma_0 \Rightarrow \alpha = 1-(1-\beta)\frac{\gamma_0}{\bar{\gamma}}$$

$$u_k = \beta u_{k-1} + \bar{\gamma}(x_k - \alpha x_{k-1})$$

$$u_k = \beta u_{k-1} + \bar{\gamma}(x_k - x_{k-1}) + (1-\beta)\gamma_0 x_{k-1}$$

Lag filter (matrix)

In order to avoid the intersampling ripple of the controlled system, for constant inputs, a constant steady-state control should be used.

The initial gain being Γ_0, and the steady state gain being $\overline{\Gamma}$, the controller will be: $u(z)=\Gamma(z)x(z)$ where

$$\Gamma(z)=\frac{1}{1-\beta z^{-1}}\overline{\Gamma}(I-\Psi z^{-1})\Rightarrow\begin{cases}\beta\rightarrow tuning\\ \Gamma(z)\xrightarrow[z=1]{}\Gamma_0\\ \Gamma(z)\xrightarrow[z\rightarrow\infty]{}\overline{\Gamma}\end{cases}$$

$$\Gamma(1)=\frac{1}{1-\beta}(I-\Psi)\overline{\Gamma}=\Gamma_0\Rightarrow\Psi=I-(1-\beta)\overline{\Gamma}^{-1}\Gamma_0$$

$$u_k=\beta u_{k-1}+\overline{\Gamma}(x_k-x_{k-1})-(1-\beta)\Gamma_0 x_{k-1}$$

Academic example

Consider the SISO plant $G(s) = \dfrac{1}{(s-0.2)(s+1.5)}$

with SS representation: $A_C = \begin{bmatrix} -1.3 & 0.6 \\ 0.5 & 0 \end{bmatrix}, \quad B_C = \begin{bmatrix} 2 \\ 0 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 1 \end{bmatrix}, \quad D = 0$

Sampling pattern: {0, 0.75}   Updating pattern: {0, 0.5}

Evolving period: T_0 =1.5 s; Fast sampling period: T=0.25 s; N=6

Fast DT plant model matrices are:

$$A = \begin{bmatrix} .7301 & 0.1285 \\ 0.1071 & 1.0084 \end{bmatrix}, \quad B = \begin{bmatrix} 0.4282 \\ 0.0282 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 1 \end{bmatrix}, \quad D = 0$$

BMIO plant model matrices are:

$$O = \begin{bmatrix} 0.2464 & 1.0633 \\ 0.366 & 1.2035 \end{bmatrix}, \quad W = \begin{bmatrix} .2896 & .43 & .7445 \\ .3316 & .2441 & .1024 \end{bmatrix}, \quad H = \begin{bmatrix} .183 & .0282 & 0 \\ .3316 & .2441 & .1024 \end{bmatrix}$$

$$P = A^6 O^\# \quad Q = W - A^6 O^\# H$$

Closed-loop behavior

The I/O controller matrices are: $U_k = MY_{d,k} + M_1 Y_{k-1} + M_2 U_{k-1}$

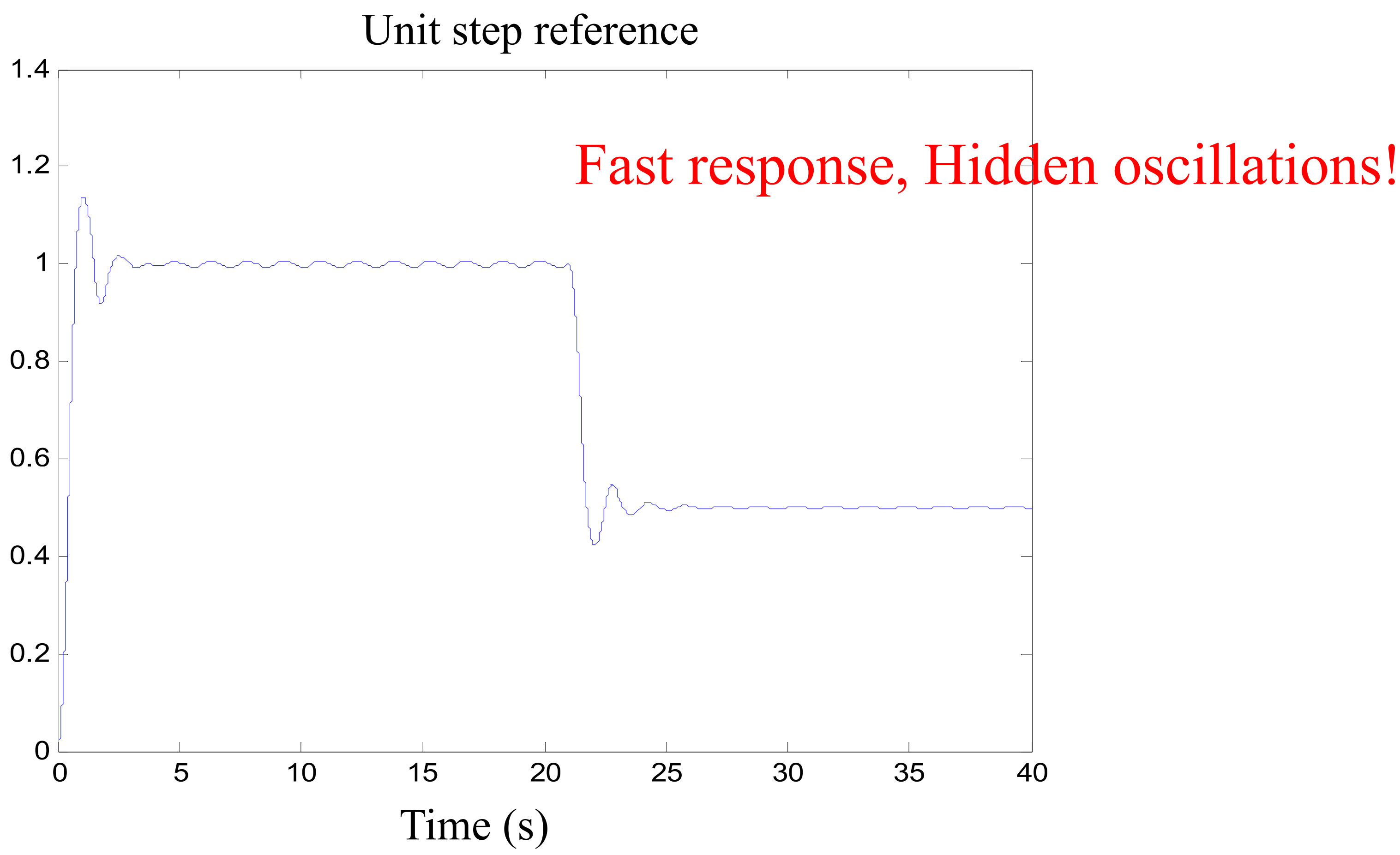
$$M = H^{\#}; \quad M_1 = -H^{\#}OP; \quad M_2 = -H^{\#}OQ$$

$$M = \begin{bmatrix} 6.5589 & -0.6190 \\ -7.1048 & 4.0213 \\ -4.3042 & 2.1836 \end{bmatrix} M1 = \begin{bmatrix} 2.1269 & -8.5164 \\ -0.4252 & 3.1924 \\ -0.3992 & 2.3968 \end{bmatrix} M2 = \begin{bmatrix} 0 & -0.0744 & -0.7893 \\ 0 & 0.0149 & 0.1578 \\ 0 & 0.0140 & 0.1481 \end{bmatrix}$$

$$\begin{bmatrix} OP + HM_1 & OQ + HM_2 \\ M_1 & M_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 2.1269 & -8.6154 & 0 & -0.0744 & -0.7893 \\ -0.4252 & 3.1924 & 0 & 0.0149 & 0.1578 \\ -0.3992 & 2.3968 & 0 & 0.0140 & 0.1481 \end{bmatrix}$$

Closed-loop poles: {0, 0, 0, 0, 0.163}

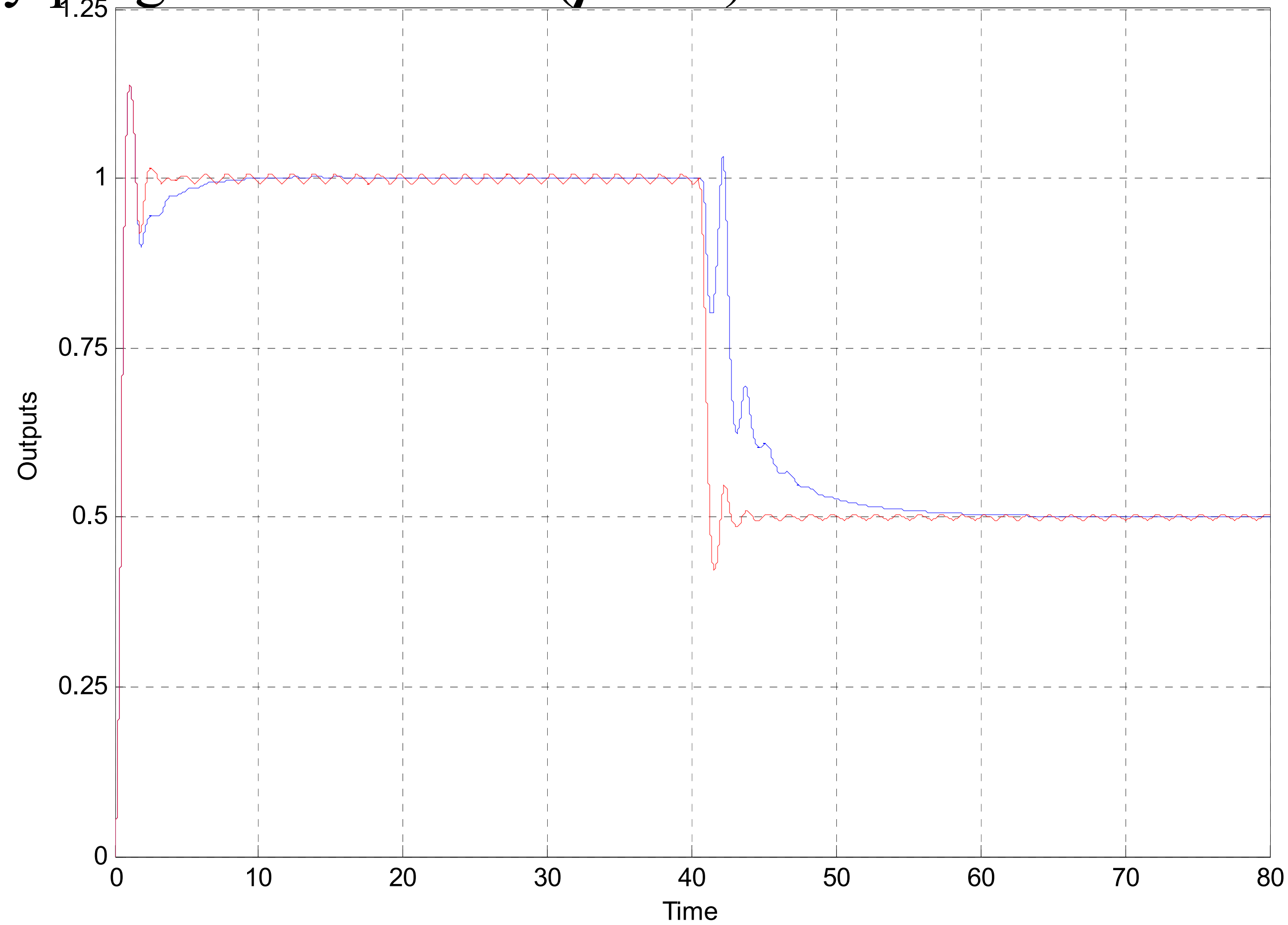
REFERENCE TRACKING



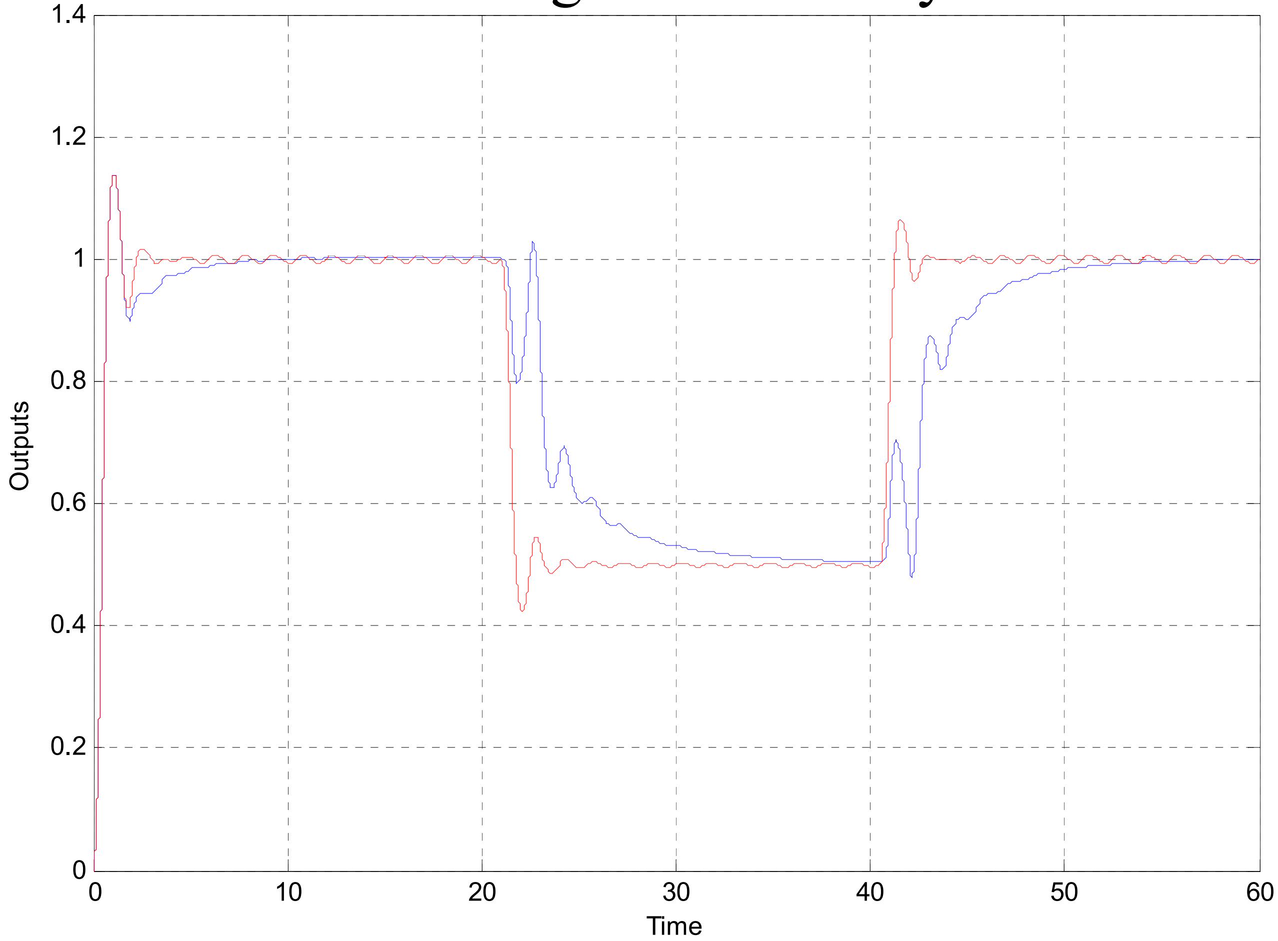
Non-uniform sampled-data control of MIMO systems. Pedro Albertos, Julián Salt
Annual Reviews in Control 35 (2011) 65–76

RIPPLE-FREE

Design Γ for stable slow behavior (poles in 0.163) and apply progressive control (β=0.7)



Controller must be reset for large errors → Hybrid Control



Lag Controller : First reply

CONCLUSIONS

- Non-Uniform Sampling/updating
  - MIMO systems
  - Instrumentation constraints
  - Computing resources optimization
- Control design
  - Get the fastest rate performance
  - Avoid intersampling ripple
- Estimate the fastest signals (output)
  - Mixed regressor
  - Kalman filter
- Ripple free control design: BMIO approach

Non-Uniformly Sampled-Data Control of MIMO Systems

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