Time Delays in Real-Time control

An overview

Pedro Albertos

pedro@aii.upv.es

Department of Systems Engineering and Control

Instituto Universitario de Automática e Informática Industrial

Universitat Politècnica de València

Time Constraints in Real Time Control - City University London. March 6, 2012

Contents

1. Introduction and Motivation

1. Introduction and motivation
   - Transport delays: Input, output, state, distributed, networking, neutral systems
   - Delays: SISO/MIMO. Single/multiple. Time invariant/variant. Uncertainty.
   - Motivation examples
2. Models and control issues
3. Classical time delayed plants control
4. Smith predictor improvements: Modified, filtered, unified SP
5. MIMO Plants.
6. Some applications
   - Aircraft Pitch control; CSTR control; Steel rolling mill; NCS
7. Conclusions and **open issues**

Introduction

1. Introduction and Motivation

- Delays are present in most industrial processes. Like non linearities.

- Delays may be caused by
  - Mass, energy or information transportation
  - Several time lags connected in series
  - Processing time of different devices: sensors, controllers, digital systems

- Long delays put some control difficulties, because
  - Disturbances are not treated immediately
  - The effect of a control action is not immediately realized
  - Current actions should take into account what has been already applied

Introduction

1. Introduction and Motivation

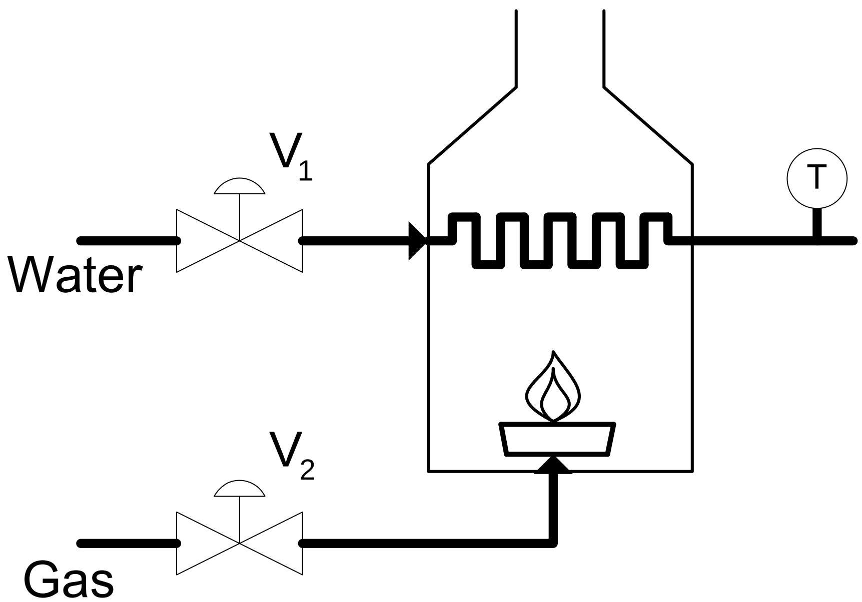
- There are different delays and they require different treatment
  - Input/output delays, due to transport or measurement processes
  - State delays, due to internal recycling of mass (energy or information)
  - Distributed delays, as in networked systems
- They can be
  - Fixed (time invariant), time variant, stochastic.
  - Single/multiple, if they appear in several points
- The process may be non linear

Examples

1. Introduction and Motivation

Water heater

- Water is heated by burning gas
- Water temperature is measured at the water outlet flow
- Measured temperature **is not** the water temperature inside the heater
- Changes in valves do not change the current water/gas flows

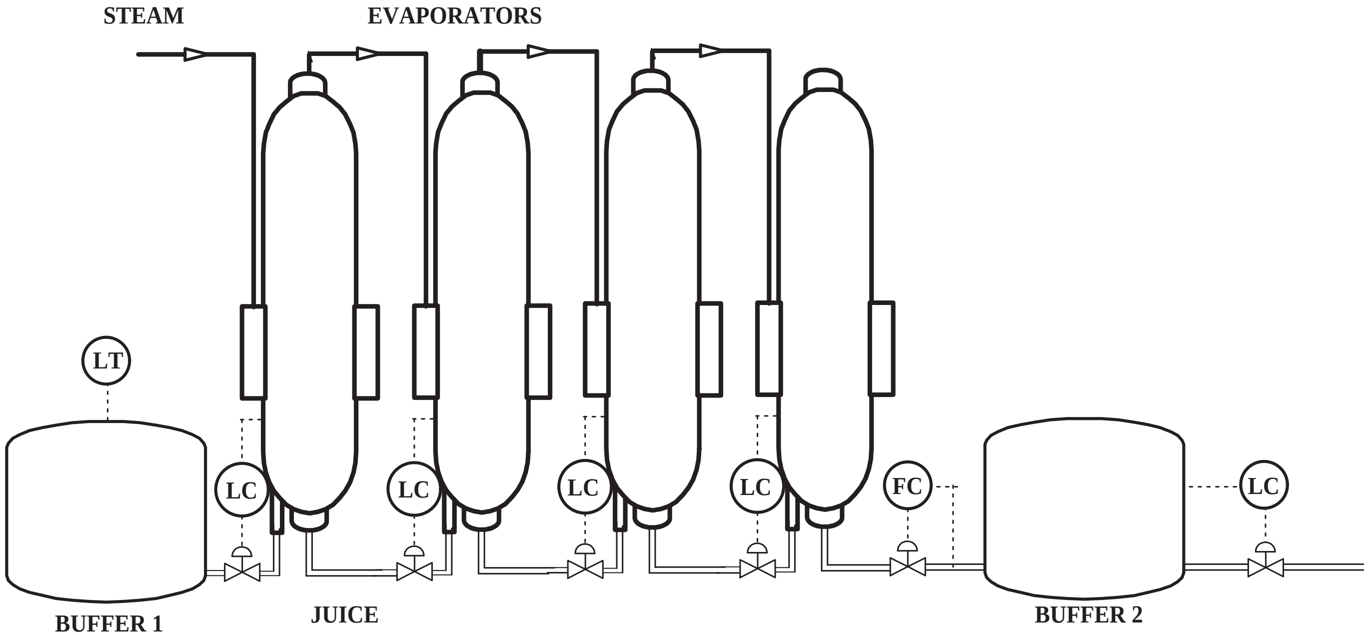


Examples

1. Introduction and Motivation

Evaporators in the sugar industry

- Steam is used to heat and evaporate the sugar cane juice
- The evaporator consists of several stages, the juice is passing through
- stages levels are locally controlled
- Changes in steam and/or juice flows have retarded effects on each stage

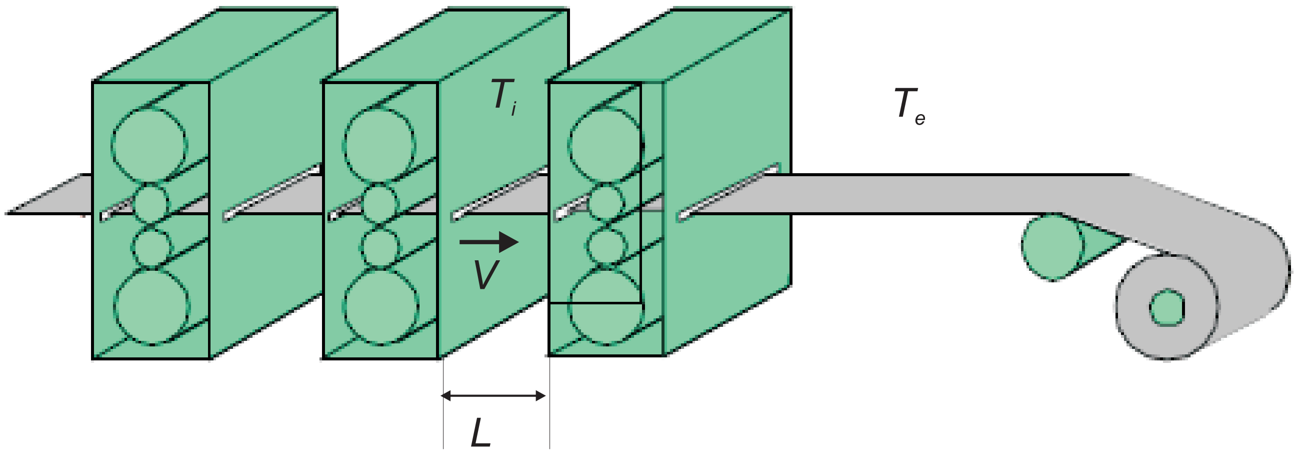


Examples

1. Introduction and Motivation

Hot Steel Rolling Mill

- Steel web is passing through different stands
- A rolling action is taken at each stand
- Interstand temperature decreases
- Thickness and temperature are measured at the exit

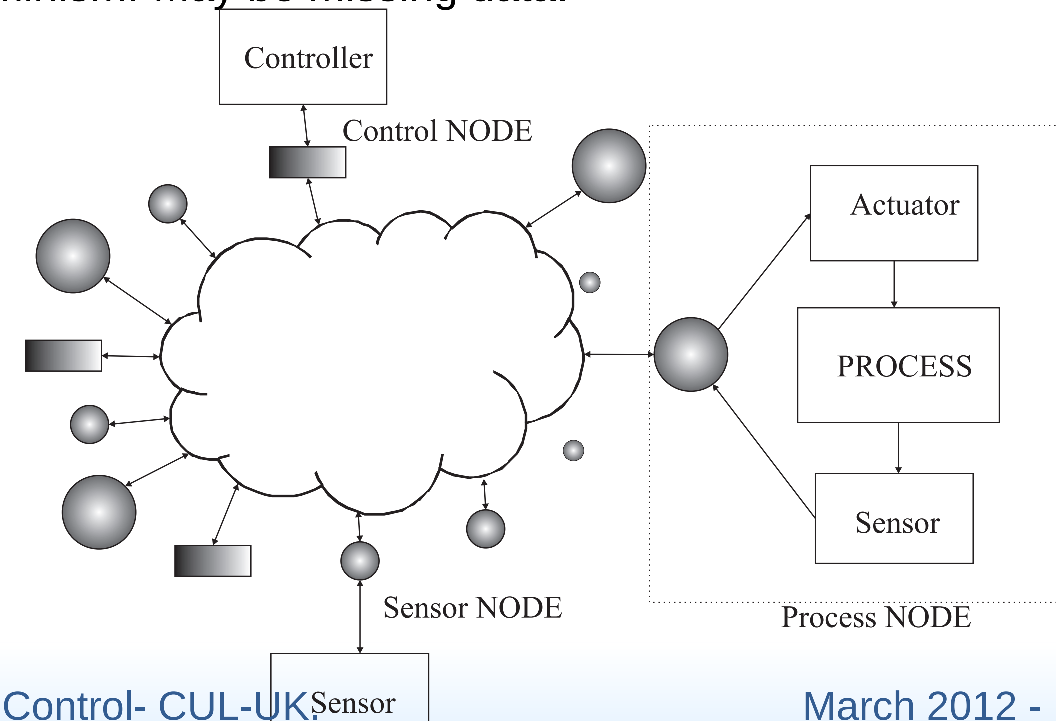


Examples

1. Introduction and Motivation

Networked Control Systems

- Process sensors and actuators are spatially distributed
- Control nodes may be located elsewhere
- Communication channels introduce non deterministic delays
- There are: network interface delay, queuing delay, transmission delay, propagation delay, link layer resending delay, transport layer ACK delay, ...$\Rightarrow$ temporal non-determinism. May be missing data.

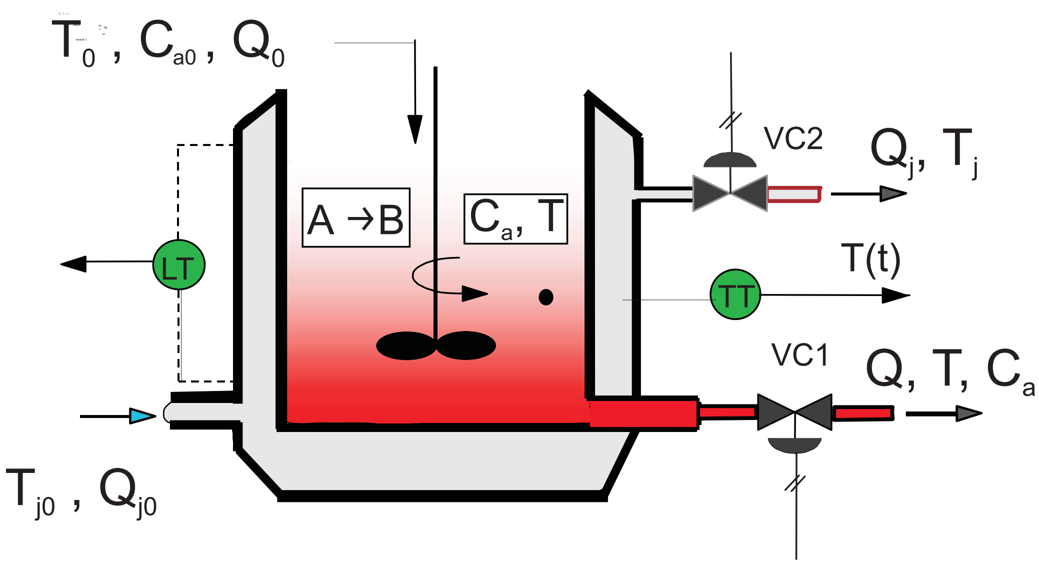


Examples

1. Introduction and Motivation

Chemical Reactor with Recycling

- Distributed process
- Delays in measuring the output variables (temperature and concentration)
- Delays in acting on the input flows (reactive and refrigerator)
- Part of the output is recycled

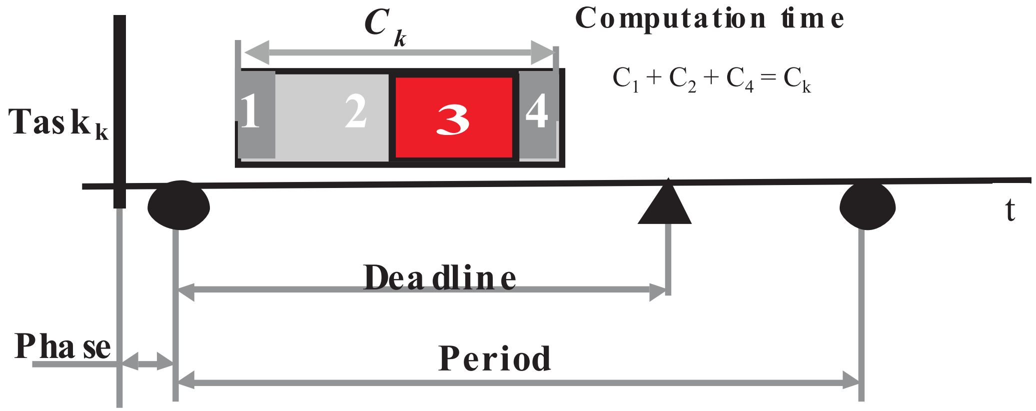


Examples

1. Introduction and Motivation

Processing delay

- CPU processes sequentially
- Data acquisition, storage and transmission
- Control action computation ⇒ Control action delivering
- Subtasks: 1) Data acquisition, 2) Mandatory (basic) control algorithm, 3) Optional (refinement) control, 4) control action delivering, 5) variables updating for next period
- Control action interval (delay + jitter)



2. Models and control issues

1. Introduction and motivation

2. Models and control issues
   - Basic Models
   - Extensions
   - Control issues
   - **Key question**: Are delays always degrading performance?

3. Classical approaches for time delayed plants control

4. Smith predictor improvements: Modified, filtered, unified SP

5. MIMO Plants.

6. Some applications
   - Aircraft Pitch control; CSTR control; Steel rolling mill; NCS

7. Conclusions and **open issues**

Basic Models

2. Models

- ***Internal representation*** Continuous time: $x \in \mathbb{R}^n; u \in \mathbb{R}^m; y \in \mathbb{R}^p$; Internal delays: r.

$$
\begin{aligned}
\dot{x}(t) &= \sum_{j=0}^{r} A_j x(t - t_j) + \sum_{i=1}^{m} b_i u_i(t - t_i) \\
y_l(t) &= \sum_{j=0}^{r_o} C_{l,j} x(t - t_j) \quad l = 1, 2, \ldots p \\
x(t_o + \theta) &= \varphi(\theta), \quad \theta \in [-\tau, 0]; \tau = \max t_j
\end{aligned}
$$

- ***External representation***

$$
\begin{aligned}
y(s) &= P(s)u(s); \quad P(s) = [p_{ij}(s)] \\
p_{ij}(s) &= g_{ij}(s)e^{-L_{ij}s}
\end{aligned}
$$

Ref: C-M. Chen. Flexible Sampling of a State Delay System. J Franklin Inst. V 334B, pp643-652, 1997.

Basic Models

2. Models

Discrete time representation: h: sampling period and, for all delays $t_i = hd_i$

Discretization of state delays in CT equation is not immediate. It is in input/output delays:

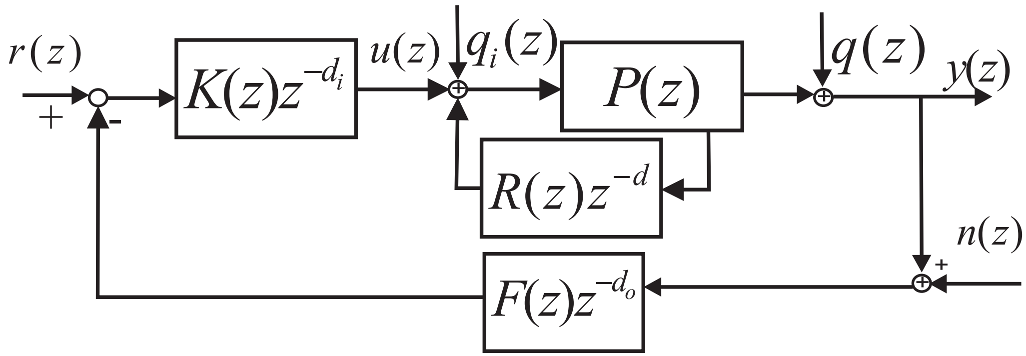
$$\dot{x}(t) = Ax(t) + A_d x(t - t_d) + bu(t) \Rightarrow t_d = dh$$

$$x_{k+1} = \bar{A}x_k + \bar{A}_1 x_{k-d+1} + \bar{A}_2 x_{k-d.} + \bar{B}u_k$$

Ref: C-M. Chen. Flexible Sampling of a State Delay System. J Franklin Inst. V 334B, pp643-652, 1997.

SISO plants

2. Models

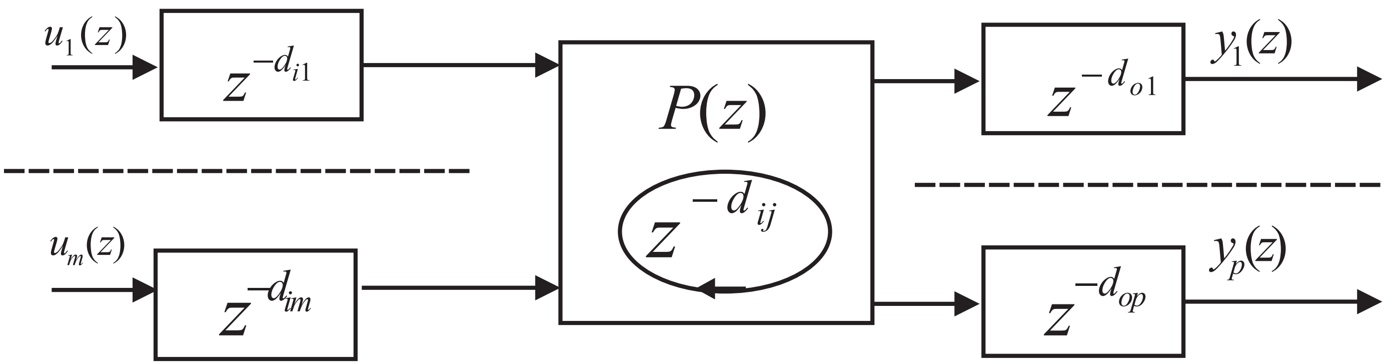


- Input delay (z^{-d_i})
- Measurement delay (z^{-d_o})
- Internal delay (z^{-d})
- Process $P(z)$, controller $K(z)$, filter $F(z)$, recycling $R(z)$
- Input/output disturbances (q)
- Measurement noise ($n(z)$)
- Loop delay ($d_i + d_o$)

MIMO plants

2. Models

- Input delay (z^{-d_i}), different for each input
- Measurement delay (z^{-d_o}), different for each output
- Internal delay (z^{-d_j}), cross-coupling delays, $j = 1, 2, \ldots$



$$y(z) = P(z)u(z); \quad P(z) = [p_{ij}(z)]$$

$$p_{ij}(z) = g_{ij}(z)z^{-d_{ij}}$$

Other delayed plant models

2. Models

- Neutral. Different delays appear in the state and its derivative

$$\dot{x}(t) = f[x(t), x(t-\tau), \dot{x}(t-\tau), u(t)]$$

- Distributed

$$\dot{x}(t) = Ax(t) + Bu(t) + \sum_{j=1}^{r} \int_{t-t_j}^{t} [G_j(\lambda)x(\lambda) + H_j(\lambda)u(\lambda)]d\lambda$$

- Combined delays

Other delayed plant models

2. Models

- Non linear. CSTR:

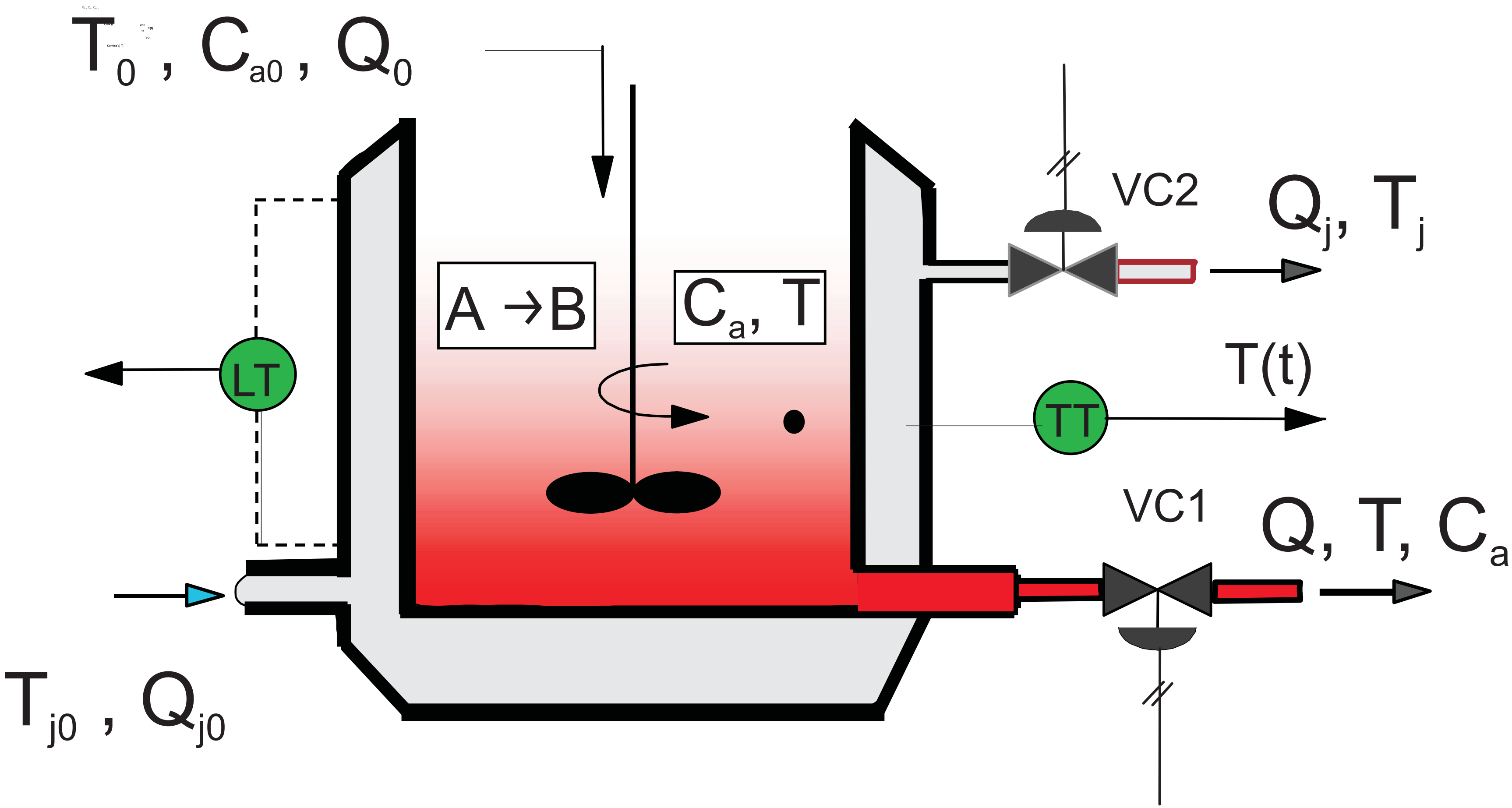
$$\frac{dC_a(t)}{dt} = \frac{(1-\lambda)q(t-L_{i,1})C_{a,0} - q(t-L_{i,1})C_a(t) + \lambda q(t-L_{i,1})C_a(t-L_x)}{V}$$
$$-\alpha C_a(t) e^{-\frac{E}{RT(t)}} \qquad (1)$$

$$\frac{dT(t)}{dt} = \frac{(1-\lambda)q(t-L_{i,1})T_0 - q(t-L_{i,1})T(t) + \lambda q(t-L_{i,1})T(t-L_x)}{V}$$
$$+\frac{H\alpha}{\rho c_p} C_a(t) e^{-\frac{E}{RT(t)}} - \frac{US}{\rho c_p V}[T(t) - T_J(t)] \qquad (2)$$

$$\frac{dT_J(t)}{dt} = \frac{q_J(t-L_{i,2})}{V_J}[T_{J,0} - T_J(t)] + \frac{US}{\rho_J c_{p,J} V_J}[T(t) - T_J(t)] \qquad (3)$$

- Uncertainty
  - Model parameters
  - Delay uncertainty
  - External disturbances

Chem. Reactor with Recycling



Control Issues

2. Models

Stability

- Let us consider the SISO case, with single state delay. The process model is:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + A_d x(t - t_d) + bu(t - t_i) \\ y(t) &= cx(t - t_o) \end{aligned}$$

- The characteristic equation is:

$$\det[sI - A - A_d e^{-t_d s}] = 0$$

The input/output delay will influence the closed-loop stability

- This can be generalized to a *quasipolynomial* characteristic equation

$$\sum_{i=0}^{n} \sum_{j=1}^{r} a_{ij} s^i e^{-t_j s} = 0$$

Tracking and regulation

2. Control Issues

- State feedback (only state delay): $u(t) = Kx(t)$, $\Rightarrow$

$$\det[sI - A - A_d e^{-t_d s} - bKe^{-t_i s}] = 0$$

- Output feedback: $u(s) = K(s)[r(s) - y(s)]$, $\Rightarrow$

$$y(s) = ce^{-t_o s}[sI - A - A_d e^{-t_d s}]^{-1} be^{-t_i s} K(s)[r(s) - y(s)]$$

Control approach

Delays are unavoidable and cannot be compensated: $\Rightarrow$

Take the delay out of the control loop (if possible) and control the “fast part" of the process.

When delays are relevant?

2. Control Issues

Delay/time constant

- If the time delay is large w.r.t. the dominant time constant
- If the "control effort" is important
- Otherwise ... delete or approximate

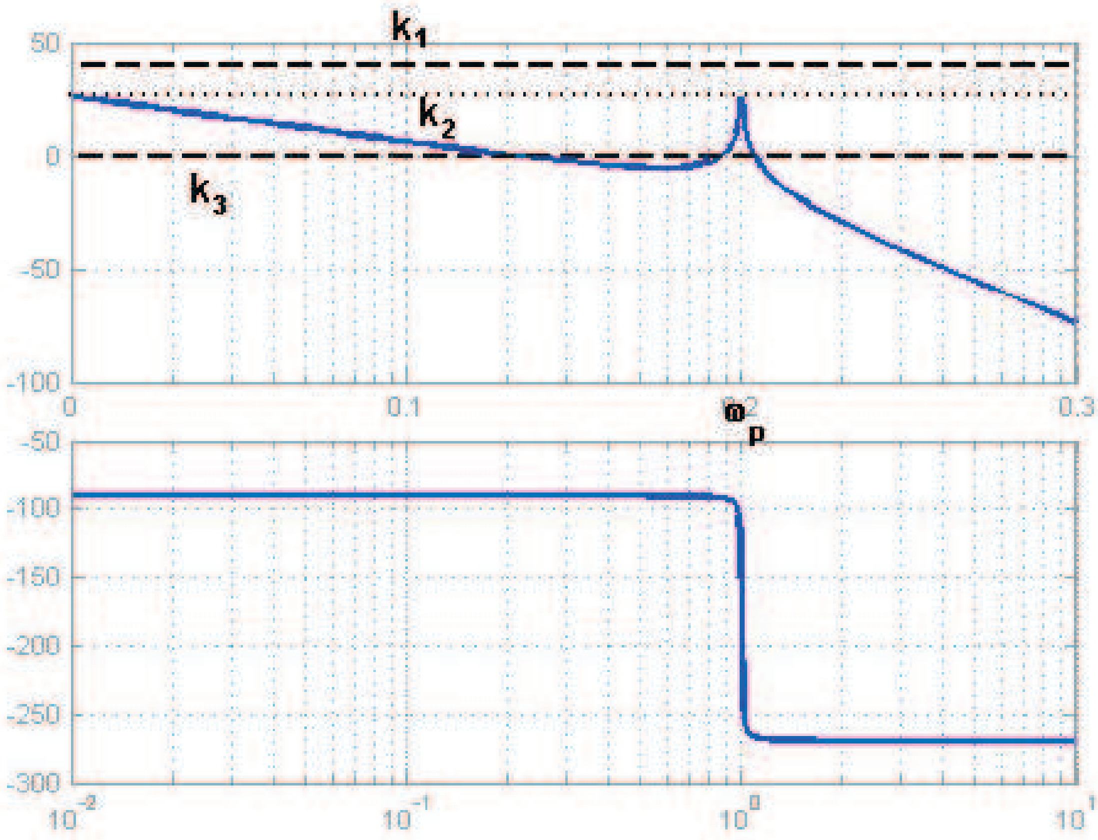
Are delays always bad?

Are delays always bad?

2. Control Issues

Resonant systems: Enlarging the bandwidth

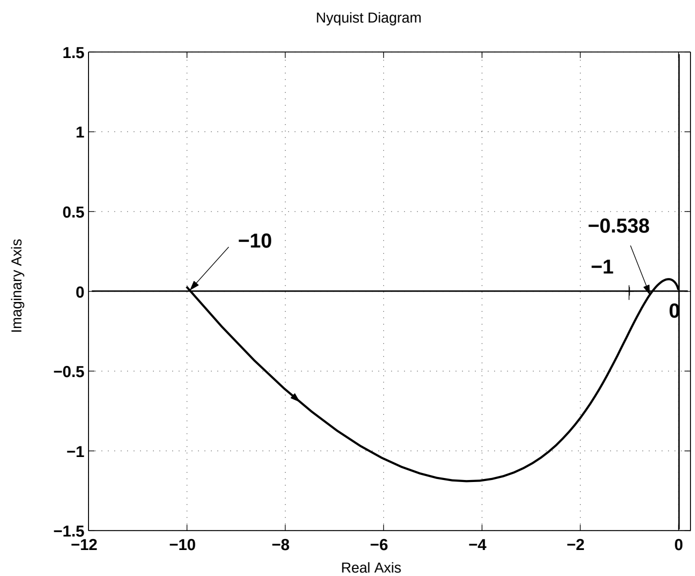
Process: $G(s) = \frac{0.01}{s(s^2+0.1s+1)}$; Controller: $K(s) = 20$; Sensor: $F(s) = e^{-\tau s}$



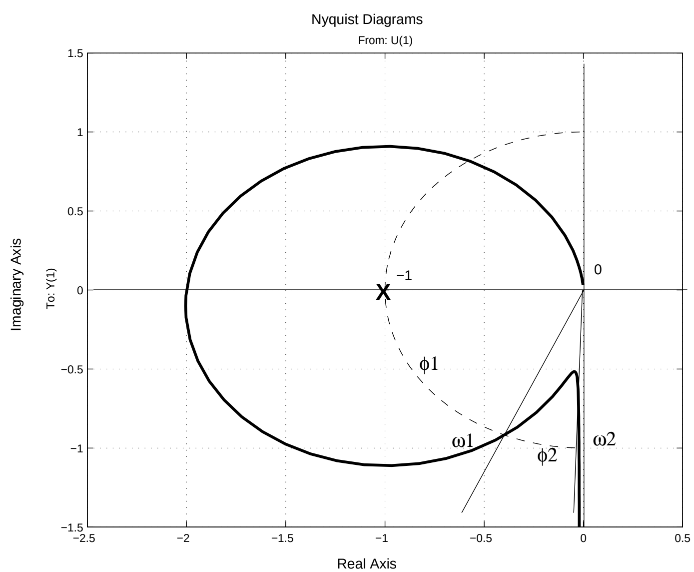
Are delays always bad?

2. Control Issues

Conditional stability



a)



b)

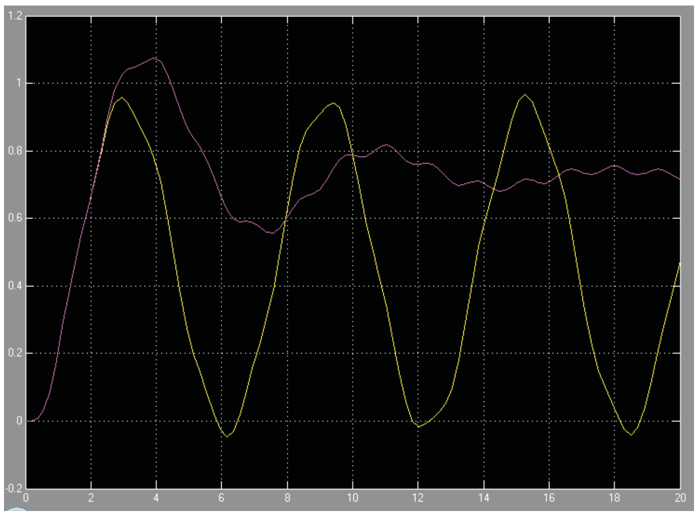
- a) Gain-conditionally stable
- b) Phase-conditionally stable

Are delays always bad?

2. Control Issues

Stabilizing delays

- Given a second order integrator, it cannot be stabilized by any static output feedback $\ddot{x}(t)u(t); \quad y(t) = x(t)$
  but it can be with $u(t) = -k_1 y(t - t_1) - k_2 y(t - t_2)$
- A double oscillator cannot be $(s^4 + (\omega_1^2 + \omega_2^2)s^2 + \omega_1^2\omega_2^2)y(s) = u(s)$ stabilized by any static output feedback but it can be with $u(t) = -ky(t - \tau)$
  as far as $sin(\omega_1 \tau) < 0$ and $sin(\omega_2 \tau) > 0$.



$k = -0.8$, $\tau = 2$, $\omega_1 = \pi + 1; \omega_2 = \pi - 1$

3. Classical TD control

1. Introduction and motivation

2. Models and control issues

3. Classical approaches for time delayed plants control
   - Time delay effect and compensation
   - Time Delay Compensator (DTC): Smith predictor (SP)
   - PID control
   - Finite spectrum assignment (FSA)
   - Model predictive control (MPC) of TD systems

4. Smith predictor improvements: Modified, filtered, unified SP

5. MIMO Plants.

6. Some applications
   - Aircraft Pitch control; CSTR control; Steel rolling mill; NCS

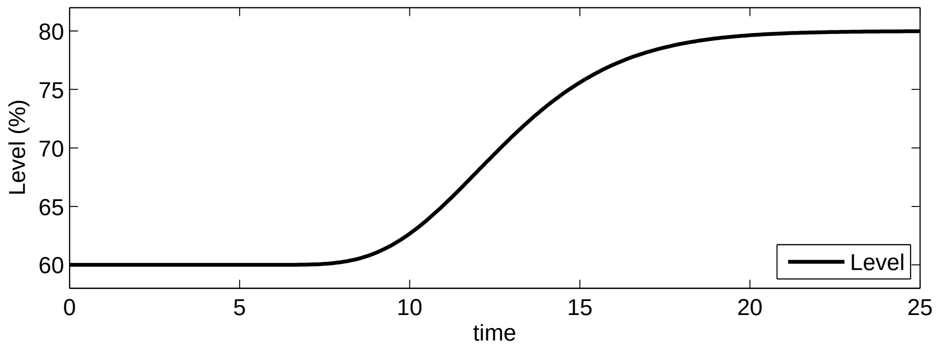
7. Conclusions and **open issues**

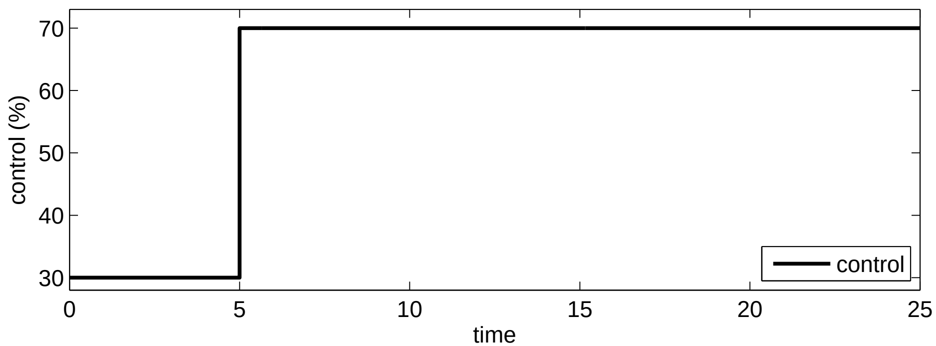
Delay effect

3. Classical TD control

SISO Input/output delay: Time response

- Step response of $G(s)e^{-sL}$, with $L > 0$.
- L represents an actual delay or an approximation



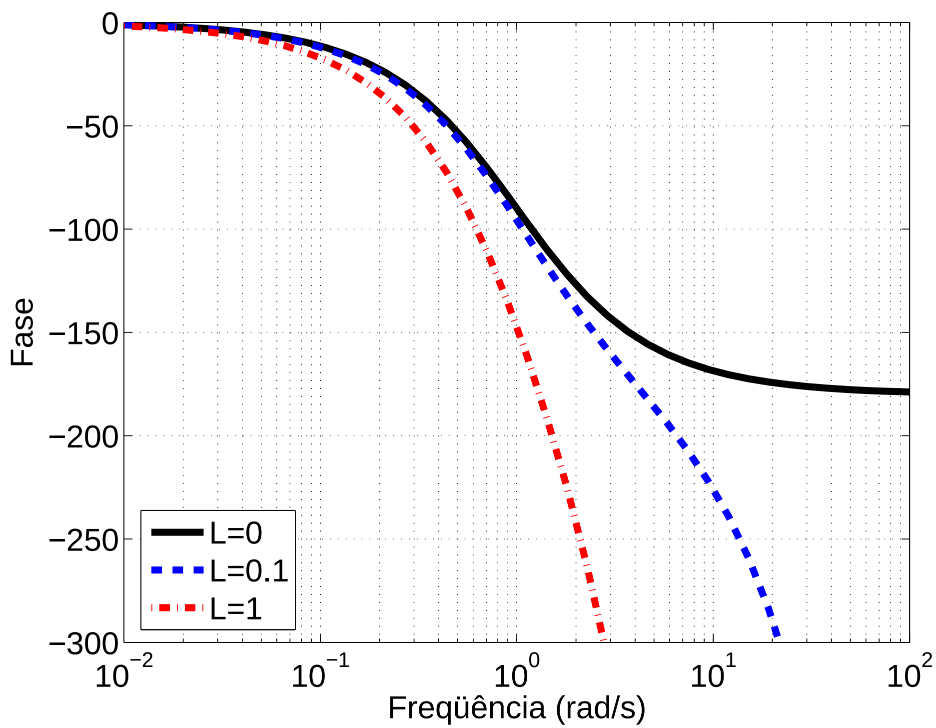


Delay effect

3. Classical TD control

SISO Input/output delay: Frequency response

- $|G(j\omega)| = |e^{-j\omega L}| = 1$, and $\angle G(j\omega) = -\omega L$
- Example $P(s) = \frac{e^{-sL}}{(1+s)^2}$

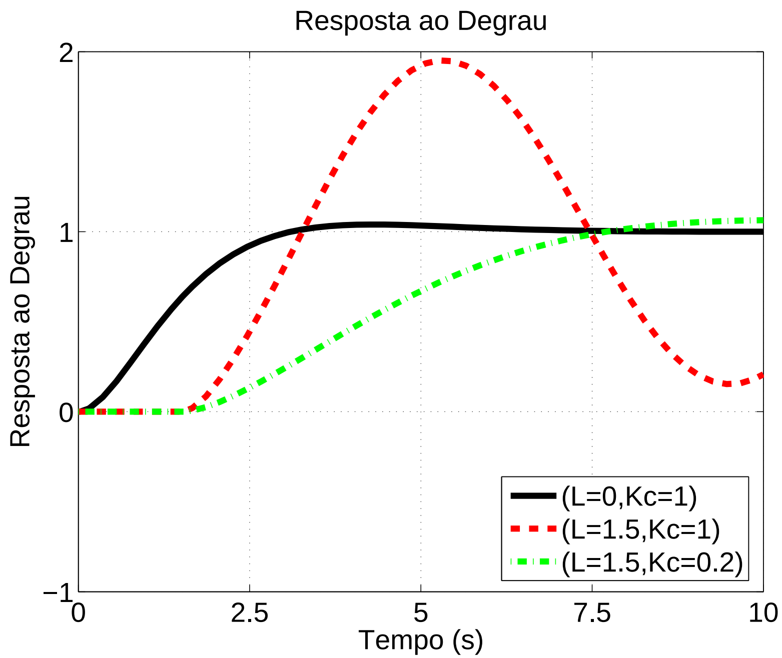


Delay effect

3. Classical TD control

SISO Input/output delay: Controlled behavior

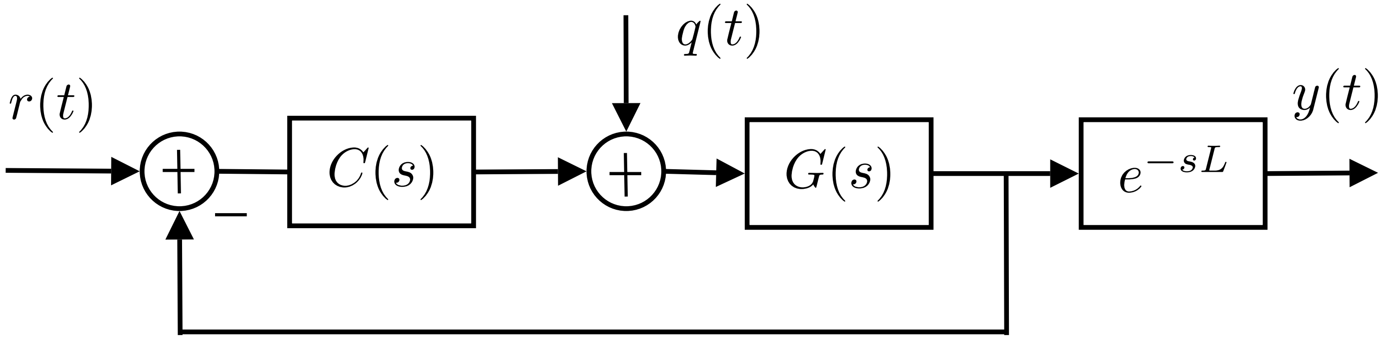
- Process $P(s) = \frac{1}{(1+1.5s)(1+0.4s)} e^{-sL}$.
- Control $C(s) = Kc \frac{(1+T_i s)}{T_i s}$.
- For $L = 0$ tuned with $Kc = 1$ e $T_i = 1.2$.
- For $L = 1.5$ reduce gain to $Kc = 0.2$.



“Ideal" control

3. Classical TD control

SISO Input/output delay: Controlled behavior



The delay is out of the characteristic equation:

$$y(s) = \frac{G(s)C(s)}{1+G(s)C(s)} e^{-sL} r(s) + \frac{G(s)}{1+G(s)C(s)} e^{-sL} q(s)$$

$$y(s) = T(s)e^{-sL} r(s) + T_d(s)e^{-sL} q(s)$$

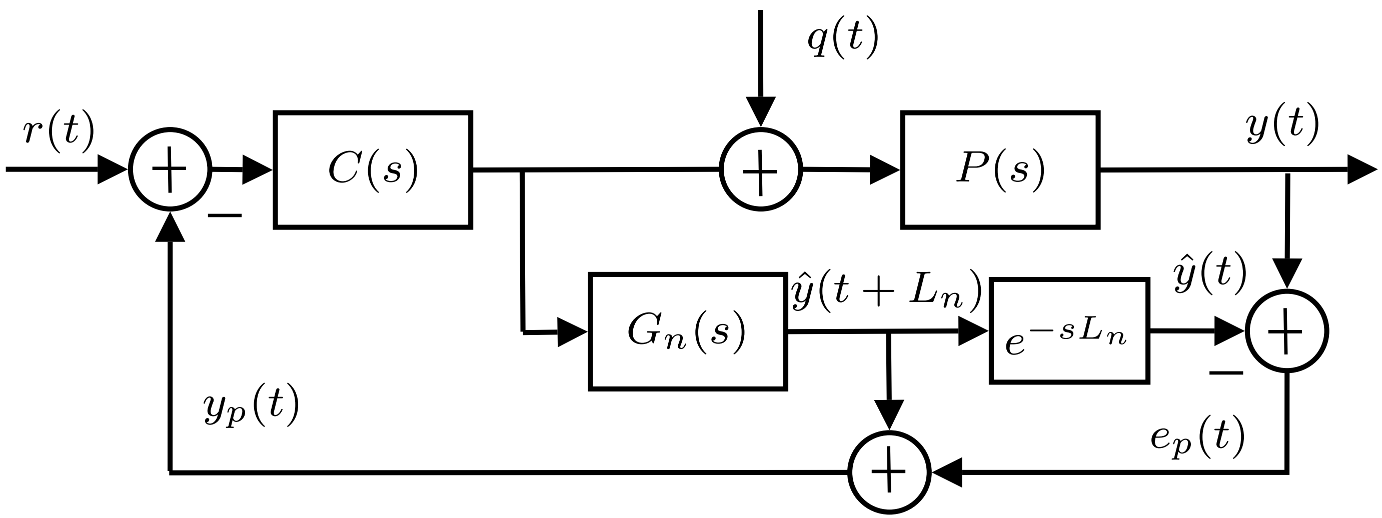
The design problem is:

- Determine $C(s)$ for $T(s)$ and $T_d(s)$
- A 2-DoF controller can be used

“Ideal" control

3. Classical TD control

Smith Predictor (SP)



Under perfect model, that is: $P(s) = P_n(s) \Rightarrow G(s) = G_n(s); L = L_n$

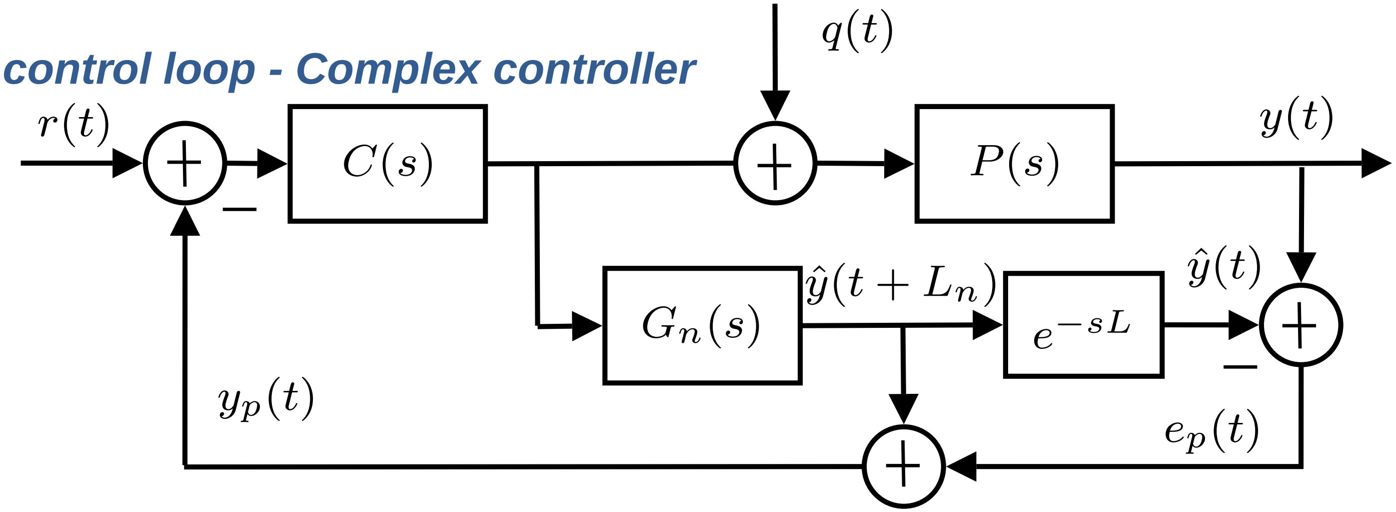
$$y(s) = T(s)e^{-sL}r(s) + G(s)e^{-sL}\left[1 - T(s)e^{-sL}\right]q(s)$$

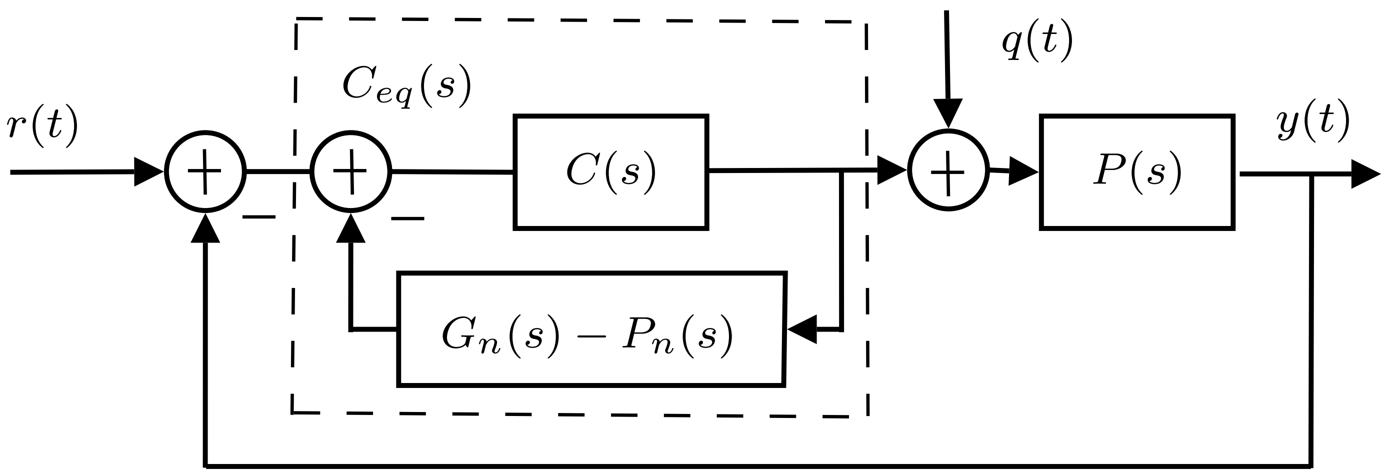
Still the delay is out of the characteristic equation, but $P(s)$ appears in $\frac{y(s)}{q(s)}$

SP equivalent controller

3. Classical TD control

Simple control loop - Complex controller





$$C_{eq} = \frac{C(s)}{1 + C(s)(G_n(s) - P_n(s))}$$

PID control

3. Classical TD control

PID models and features (review)

- Basic PID

$$C(s) = K_c(1 + \frac{1}{T_i s} + T_d s) \quad \text{(series)}$$

$$C(s) = \frac{K_c(1 + \frac{1}{T_i s} + T_d s)}{\alpha T_d s + 1} \quad \text{(proper)}$$

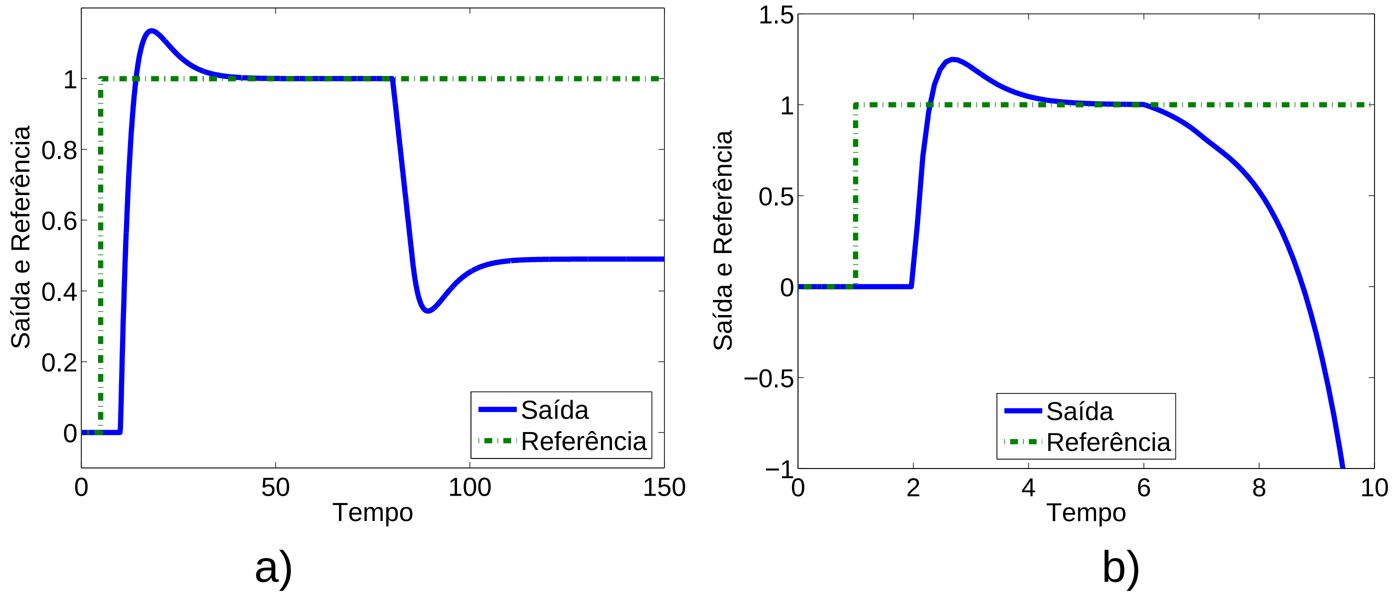
- α is tuned for noise cancelation and robustness, $\alpha \in (0, 1)$.
- Other implementations:

$$C(s) = K_c \frac{1 + T_i s}{T_i s} \frac{T_d s + 1}{\alpha T_d s + 1} \quad \text{(series)}$$

$$C(s) = K_p + \frac{K_i}{s} + \frac{K_d s}{\alpha K_d s + 1} \quad \text{(parallel)}$$

PID control: Drawbacks

3. Classical TD control



a) Steady-state error Process $P(s) = \frac{2e^{-5s}}{s(1+0.1s)}$ model $P_n(s) = \frac{2e^{-5.1s}}{s}$.

Primary controller PI: $C(s) = 0.25\frac{8s+1}{8s}$. Step disturbance -0.05 at $t = 75$.

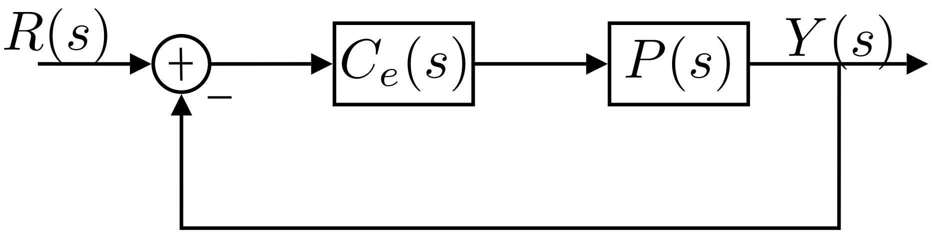
b) Internal instability Process $P(s) = \frac{e^{-s}}{(s-1)}$ PI primary controller with $K_c = 6$, $T_i = 1$.

Closed-loop poles assigned at $s = -2$ e $s = -3$. Step disturbance -0.1 at $t = 5$.

Approximated PID control

3. Classical TD control

Use Padé approximation: $P(s) = \frac{K}{1+Ts}e^{-sL}$; $e^{-sL} \approx \frac{1-0.5Ls}{1+0.5Ls}$



1. Design a PID for: $P_n(s) = \frac{K}{1+Ts}\frac{1-0.5Ls}{1+0.5Ls}$
2. Design a controller PI $C(s) = \frac{K_1(1+sT_1)}{sT_1}$ for $G(s) = \frac{K}{1+Ts}$
   - The SP equivalent controller is: $C_e = \frac{C(s)}{1+C(s)[Gn(s)-Pn(s)]}$
   - If $T_1 = T$ results: $C_e(s) = \frac{K_1(1+Ts)}{Ts+K_1K(1-e^{-sL})}$
   - The controller is:

$$C_e(s) = K_c \frac{1+T_i s}{T_i s} \frac{T_d s + 1}{\alpha T_d s + 1}$$

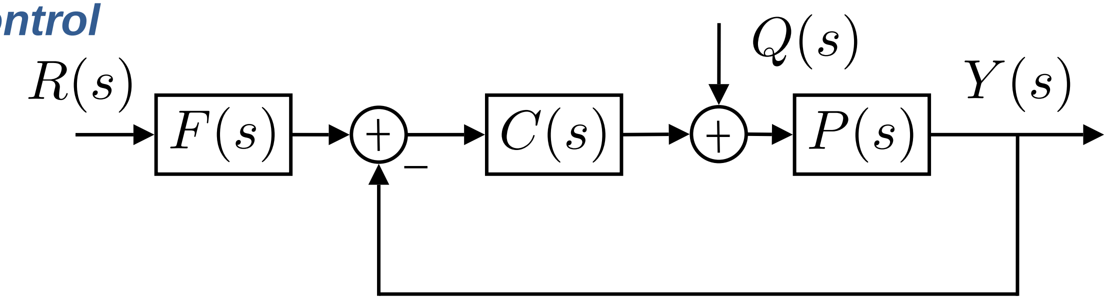
with: $K_c = \frac{T}{(L+T_0)K}$, $T_i = T$, $T_d = 0.5L$, $\alpha = \frac{T_0}{T_0+L}$

To get a closed-loop time constant $T_0 = \frac{T}{KK_1}$.

PID control extensions

3. Classical TD control

2-DoF PID control

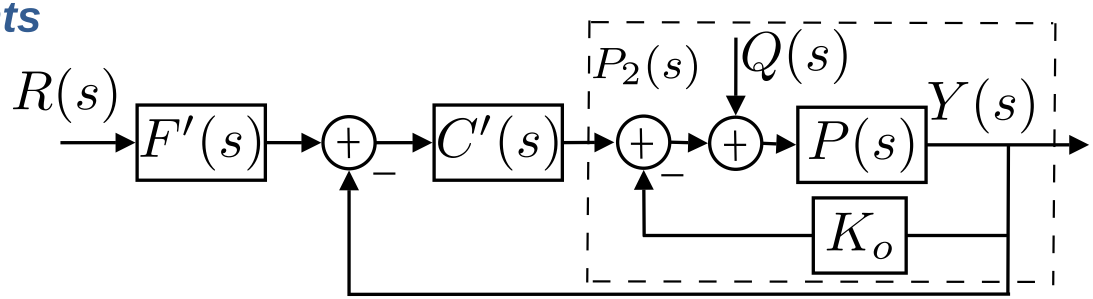


- 2-DoF structure:

  - closed-loop poles for disturbance rejection: $C(s) = K_c \frac{(1+T_i s+T_i T_d s^2)}{T_i s(T_f+1)}$

  - set-point filter to smooth the reference response: $F(s) = \frac{1+bT_i s+cT_i T_d s^2}{1+T_i s+T_i T_d s^2}$

Unstable plants



- Sequence: 1) Stabilizing controller K_o; 2) PID for equivalent plant P_2; 3) Set-point filter

PID tuning

3. Classical TD control

Dominant delay

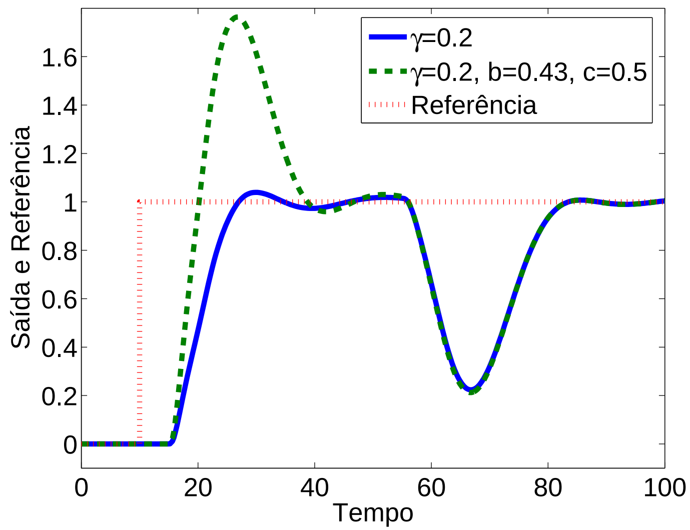
- Stabilize $P(s)$ with simplest P controller: K_0
- Approximate $P_2 = \frac{Ke^{-sL}}{(1+Ts)}$
- Use PID controller $C(s) = K_c \frac{(1+Ts)(1+0.5Ls)}{Ts(1+0.5\alpha Ls)}$
- Define the closed-loop time constant T_0 as a function of L: $\frac{T_0}{L} = \frac{\alpha}{1-\alpha}$
- For each α, the nominal response is the same (time scaled by L).
- The greater α is the more robust the system results, and the slower it is!
- Define the set-point filter $F(s)$.

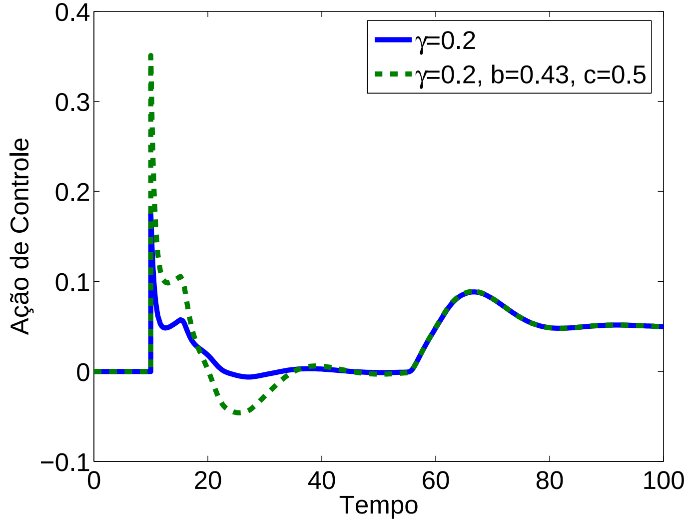
PID control example

3. Classical TD control

Design example

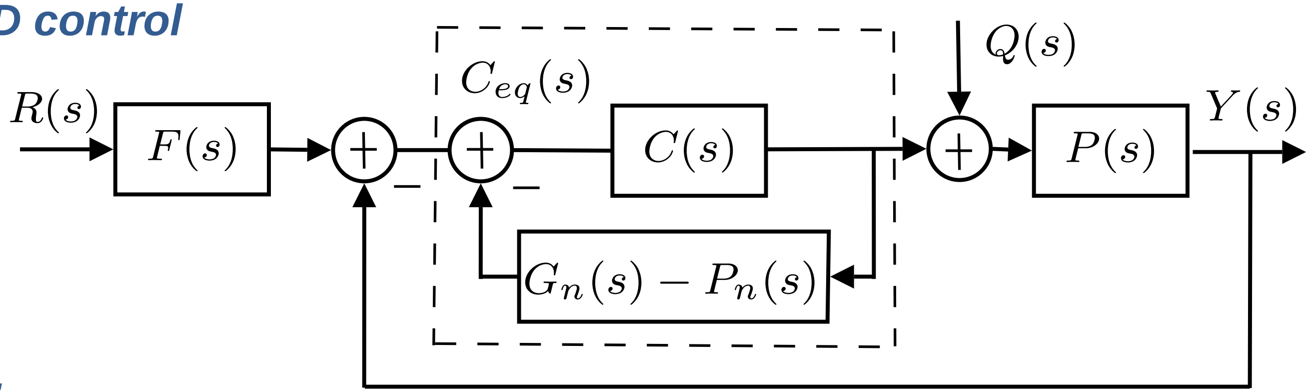
- Process $P(s) = \frac{0.2e^{-5s}}{s(1+s)(1+0.5s)(1+0.1s)}$, Model $P_n(s) = \frac{0.2e^{-5s}}{s}$.
- tune $K_0 = 0.3$ to stabilize $P_2 \approx \frac{0.33e^{-5s}}{(1+20s)}$
- Tuning for $\alpha = 0.2$ Case 1: $b = c = 1$, Case 2: $b = 0.43$; $c = 0.5$.





PID control example

3. Classical TD control



2DoF-SP.

- For $P_n = \frac{K_p e^{-sL_n}}{1+Ts}$ and $G_n = \frac{K_p}{1+Ts}$:
  - Choose $C(s) = K_c(1 + \frac{1}{sT_i})$, with $T_i = T$: $\quad 1 + C(s)G_n(s) = 1 + \frac{k_c K_p}{sT_i}$
  - Define the required closed-loop time constant T_0, by: $K_c = \frac{T_i}{KpT_0}$
  - Tune the input filter $F(s) = \frac{1+sT_0}{1+sT_1}$
- This allows dealing with tracking (T_1) and disturbances (T_0):

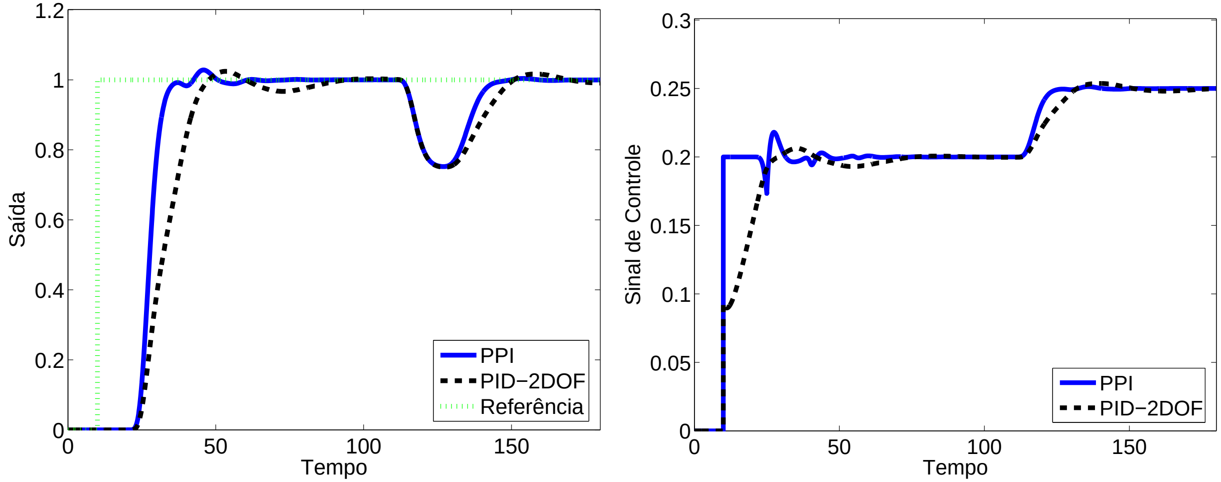
$$\frac{Y(s)}{R(s)} = \frac{e^{-sL_n}}{1+sT_1} \quad \& \quad \frac{Y(s)}{Q(s)} = \frac{K_0 e^{-sL_n}}{1+sT}\left[1 - \frac{e^{-sL_n}}{1+sT_0}\right]$$

- Simple tuning for delay dominant systems: $T_0 = T_1 = T \Rightarrow K_c = \frac{1}{K_p}$

(*Predictive PI Controller*)

2DoF-SP PID control

3. Classical TD control



- Plant: $P(s) = \frac{5e^{-10s}}{(1+s)^8}$; Model: $Pn(s) = \frac{5e^{-15s}}{1+3s}$.
- First tune a PID with $\alpha = 0.3$ and then:

$K_c = \frac{0.35(L+2T)}{K_p L}$	$T_i = T + 0.5L$	$T_d = \frac{LT}{L+2T}$	$T_f = 0.15L$	$b = 0.80$	$c = 1$

- As a result: PPI is faster and less oscillatory.
- In general, the SP presents a good tradeoff between performance and robustness.

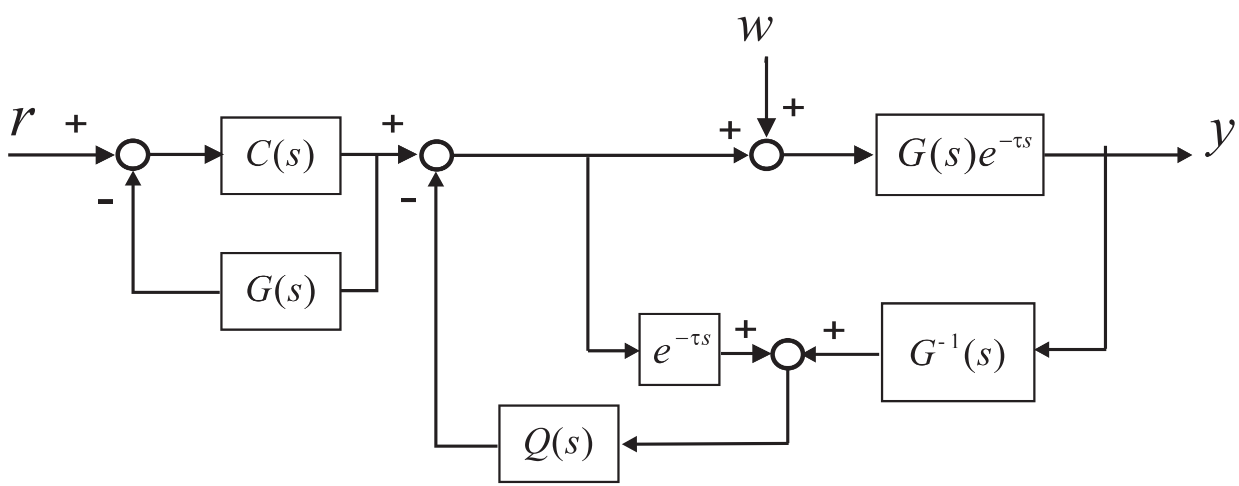
4. SP improvements

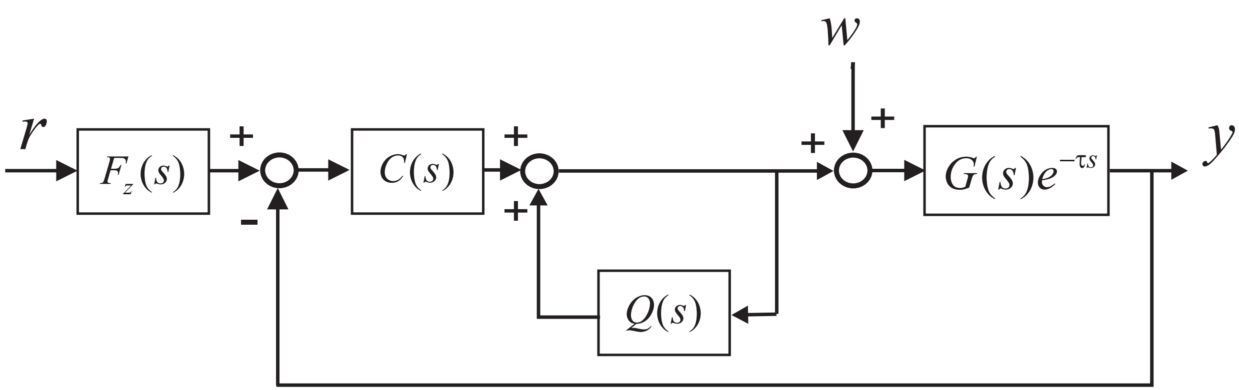
Modified SP

4. SP Improvements

Proposed by Zhong

A generalized controller $Q(s)$ is added:

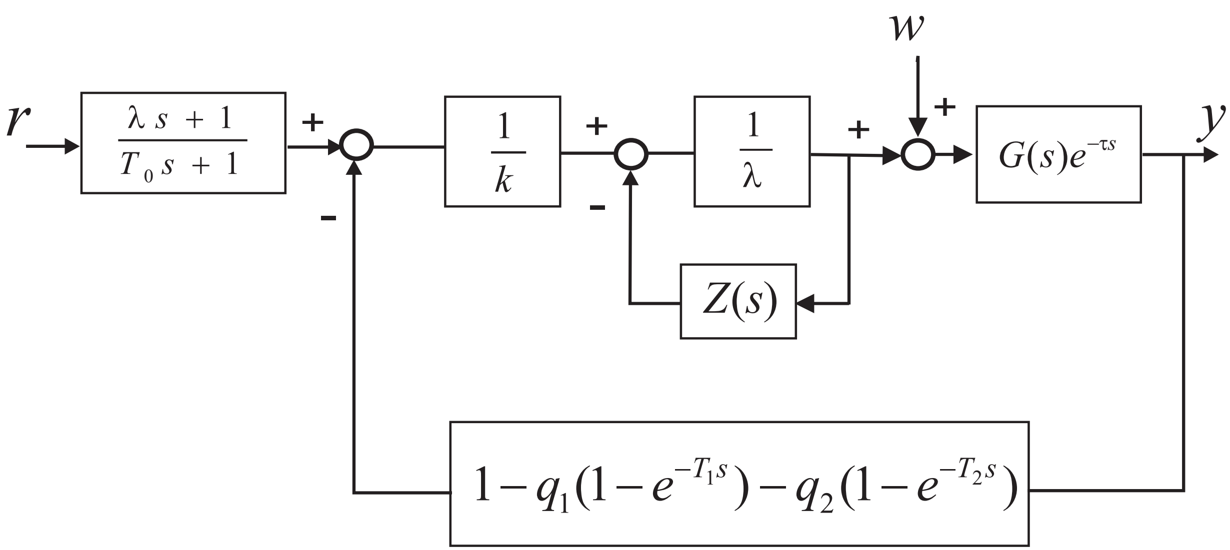




Modified SP

4. SP Improvements

Zhong implementation example

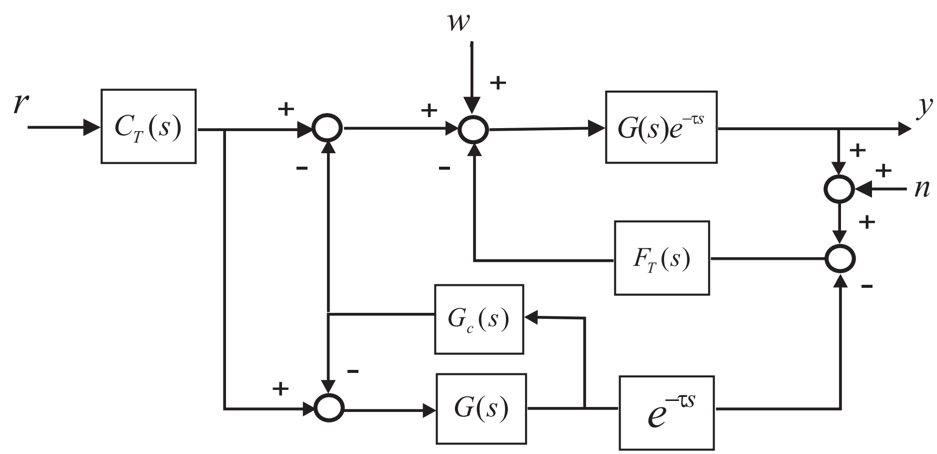


Change

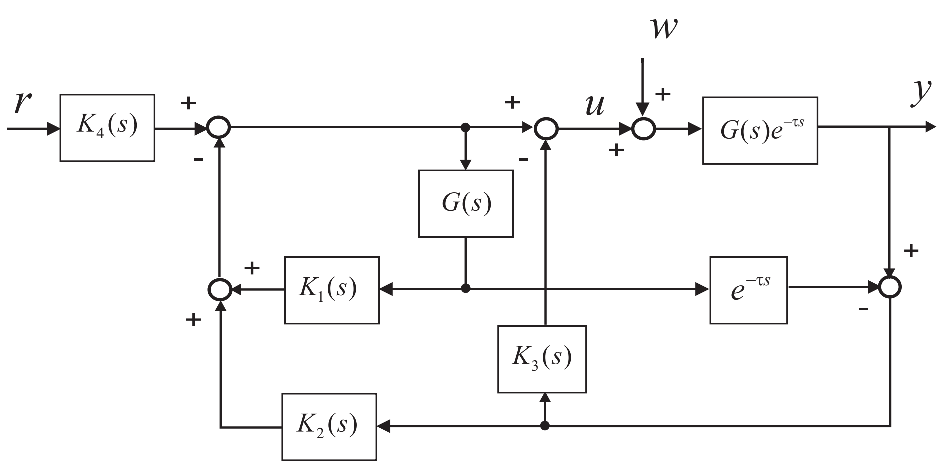
Modified SP

4. SP Improvements

Proposed by Tao: Rejecting load disturbances



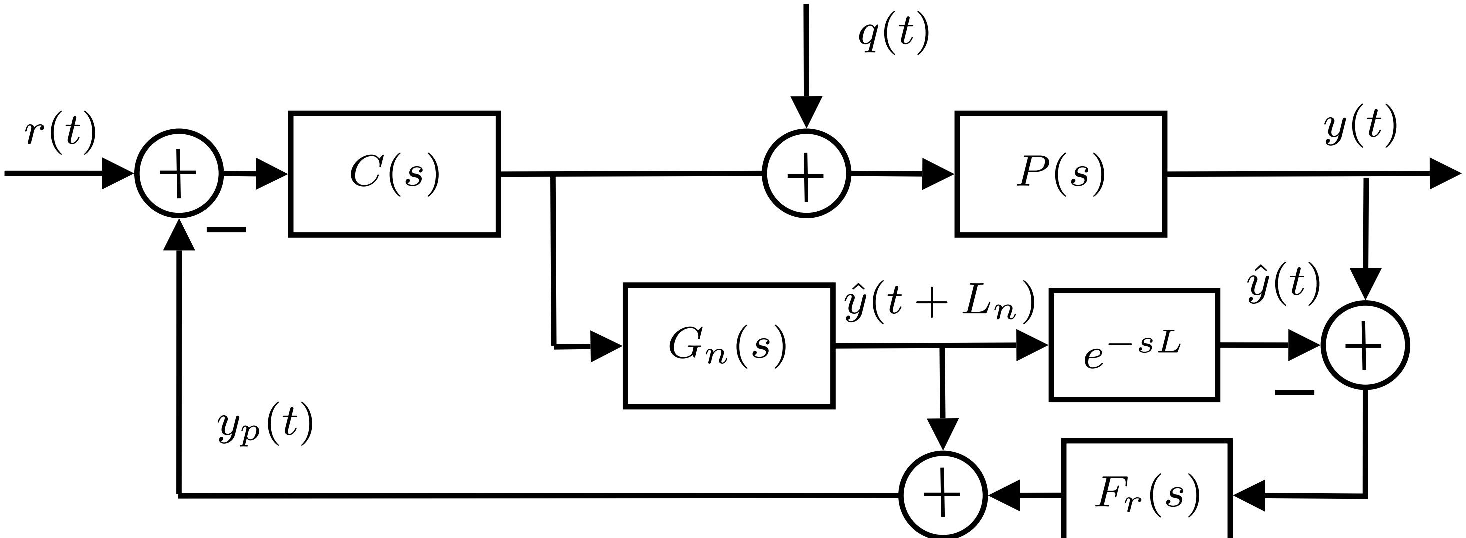
Proposed by Xiang: Dealing with integrative processes

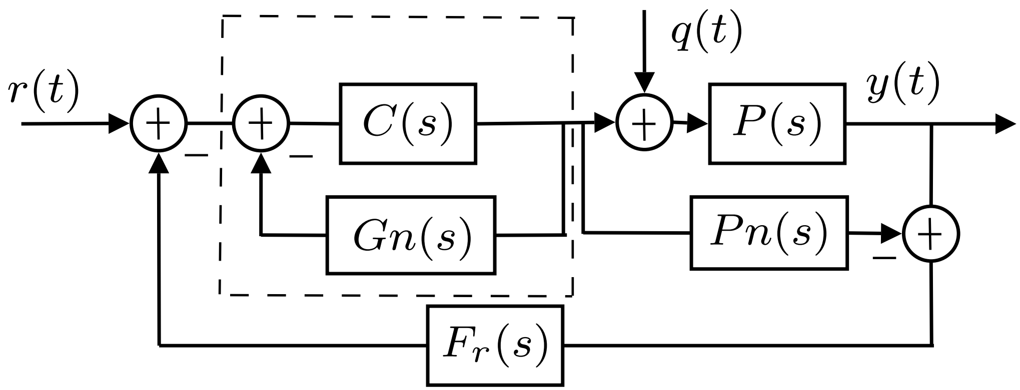


Filtered SP

4. SP Improvements

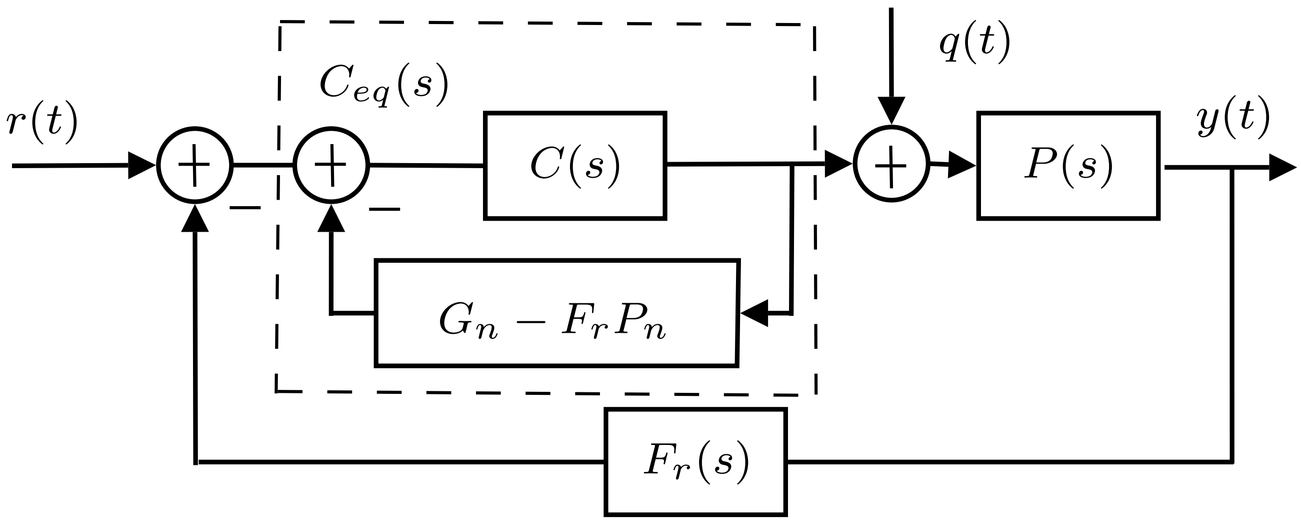
[Normey-Rico]: SP with prediction filter $F_r(s)$, to reduce modeling errors:





FSP: Equivalent controller

4. SP improvements



$$\frac{Y(s)}{R(s)} = \frac{C_{eq}(s)P(s)}{1 + C_{eq}(s)F_r(s)P(s)} \quad \& \quad \frac{Y(s)}{R(s)} = \frac{C_{eq}(s)}{1 + C_{eq}(s)F_r(s)P(s)}$$

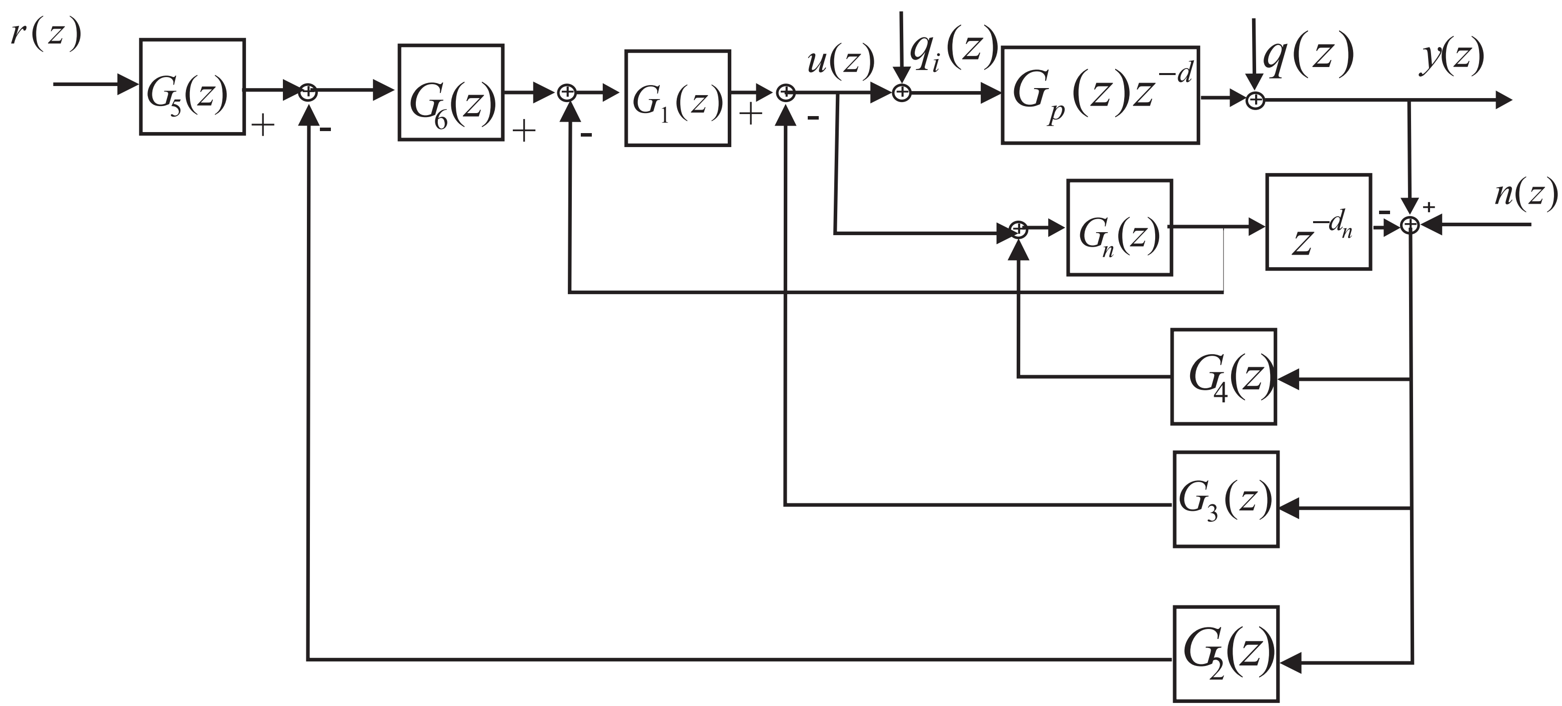
- F_r should be “low-pass"
- Avoid low time constant cancelation
- Allows disturbance rejection

A global setting of the SP

4. SP Improvements

Most proposals can be fitted in this schema

- If $G_1 = G_6 = G_5 = 1$
- G_3 applies for servo; G_4 applies for regulation and G_2 applies for model mismatch.



- Difficult tuning of the many controllers.

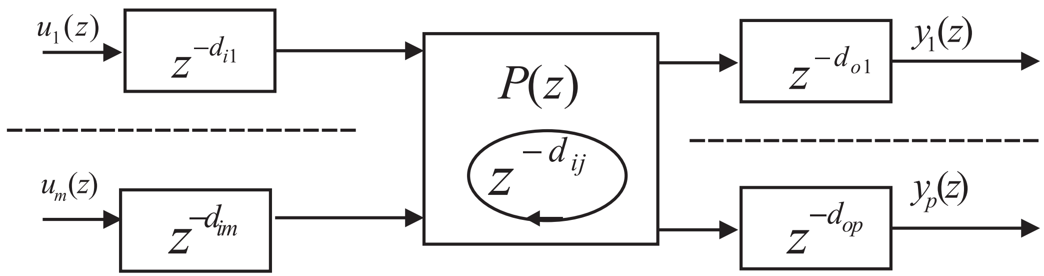
5. MIMO Plants

1. Introduction and motivation
2. Models and control issues
3. Classical time delayed plants control
4. Smith predictor improvements
5. MIMO Plants.
   - A SISO stable output predictor. GP robustness properties.
   - Multiple delays. Extension to MIMO plants
   - MIMO plants decoupling
   - **A complete solution?**
6. Some applications
   - Aircraft Pitch control; CSTR control; Steel rolling mill; NCS
7. Conclusions and **open issues**

Delayed MIMO plants

5. MIMO plants

$$y(z) = P(z)u(z); \quad P(z) = [g_{ij}(z)z^{-d_{ij}}]$$



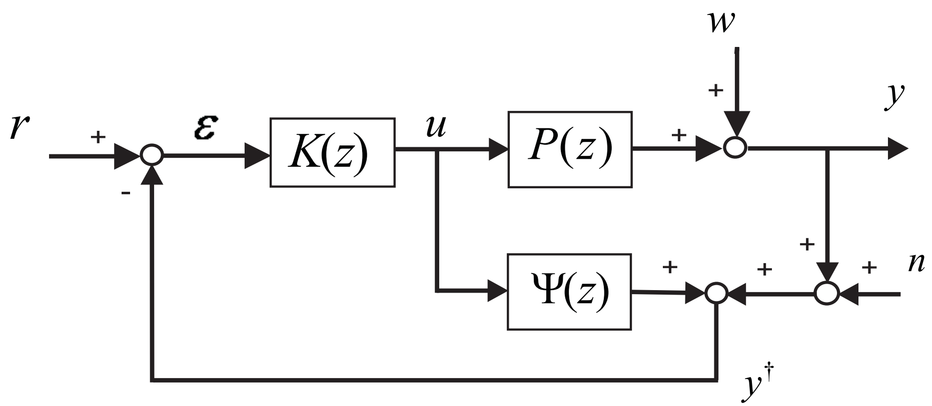
1. Measurement delay ($z^{-d_{j,o}}$), different for each output: $P(z) = D(z)G(z)$ where $D(z)$ is a diagonal matrix of delays and $G(z)$ is the “fast" model of the plant. SP can be used with $P_n(z) = D_n(z)G_n(z)$
2. Input delay ($z^{-d_{j,i}}$), different for each input. Now $P(z) = G(z)D(z)$ and the only option is to increment the delays at the required inputs to get a common delay. Then $P_n(z) = z^{d_n}G_n(z)$ and SP is applicable
3. Internal delay (z^{-d_j}), cross-coupling delays, $j = 1, 2, \ldots$. Classical SP is not applicable anymore

A Generalized Predictor

SISO plants

- For SISO I/O delayed plant: $P_p(z) = g_p(z)z^{-d_p}$

$$x_{k+1} \quad = \quad Ax_k + bu_k; \quad y_k = cx_k; \quad g(z) = c(zI - A)^{-1}b = \frac{n(z)}{d(z)}$$



- Define:

$$y^\dagger(z) := \psi_d(z)u(z) + y(z) = g^\dagger(z)u(z); \quad g^\dagger = \frac{n^\dagger(z)}{n(z)}g(z) = cA^d(zI - A)^{-1}b$$

$$\psi_d(z) = cA^{-d}\sum_{i=1}^{d} A^{i-1}bz^{-i}$$

GP for MIMO plants

5. MIMO plants

$$G(z) = C(zI - A)^{-1}B = \begin{bmatrix} c_1 \\ \vdots \\ c_m \end{bmatrix} (zI - A)^{-1} \begin{bmatrix} b_1 & \cdots & b_m \end{bmatrix}$$

Define:

$$\Psi(z) := \begin{bmatrix} \psi_{11}(z) & \cdots & \psi_{1m}(z) \\ \vdots & \ddots & \vdots \\ \psi_{m1}(z) & \cdots & \psi_{mm}(z) \end{bmatrix}; \quad \psi_{ij}(z) = c_i A^{-d_{ij}} \sum_{k=1}^{d_{ij}} A^{k-1} b_j z^{-k}$$

This leads to: $G^\dagger(z) = \begin{bmatrix} g_{11}^\dagger(z) & \cdots & g_{1m}^\dagger(z) \\ \vdots & \ddots & \vdots \\ g_{m1}^\dagger(z) & \cdots & g_{mm}^\dagger(z) \end{bmatrix} \equiv (\Psi(z) + P(z))$

$$g_{ij}^\dagger(z) \equiv \psi_{ij}(z) + p_{ij}(z)$$

Design procedure

5. MIMO plants

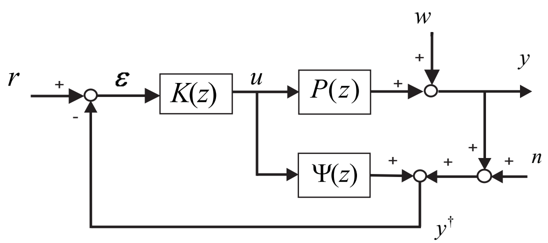
.

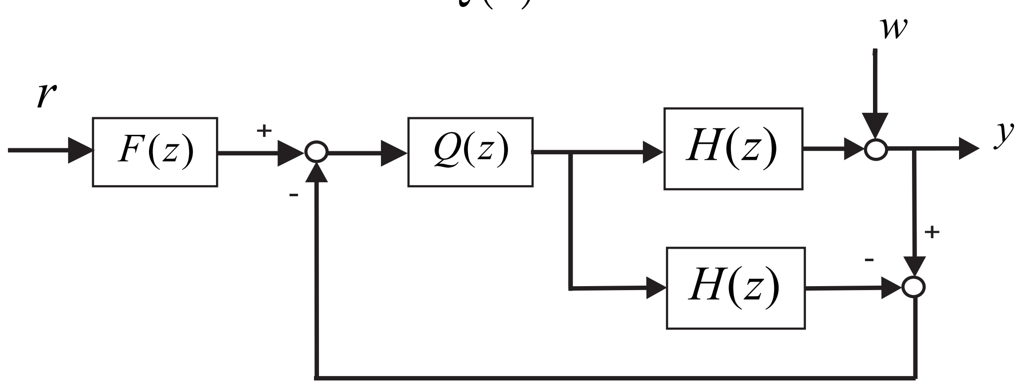
Given the DT full plant model, $P(z)$

- Compute the rational transfer matrix $G(z)$,
- Get the undelayed output transfer matrix $G^\dagger(z)$,
- Compute the predictor filter $\Psi(z)$,
- Stabilize the MIMO system by controller $K(z)$, for $G^\dagger(z)$, such that

$$H(z) = P(z)K(z)[I + G^\dagger(z)K(z)]^{-1}$$

- For the previously stabilized plant, $H(z)$, improve the controlled system performance by, a robust performance controller $Q(z)$.

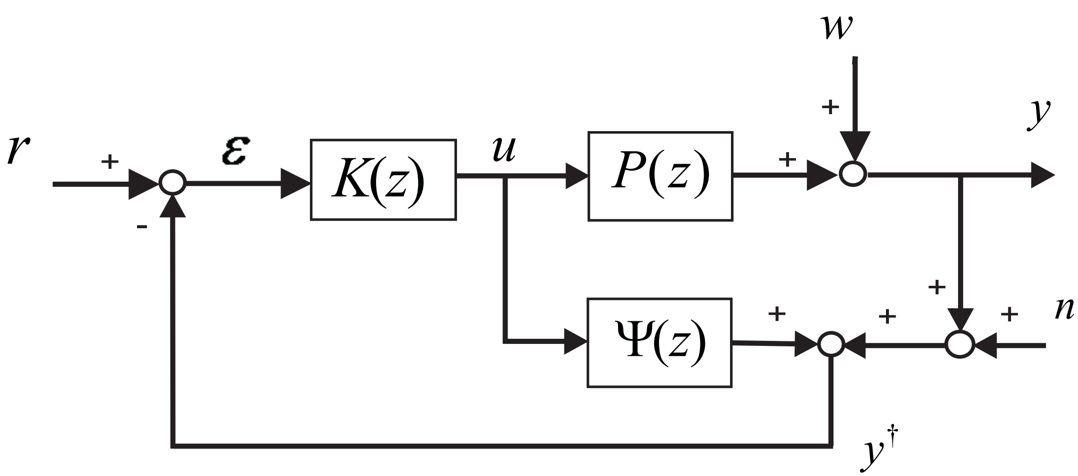




Design example

5. MIMO plants

- Plant model, $P(s)$, $h = 0.1s$



$$P(s) = \begin{bmatrix} \frac{e^{-0.5s}}{s-1} & \frac{0.5e^{-0.7s}}{s+1} \\ \frac{0.1e^{-0.3s}}{10s+1} & \frac{e^{-0.7s}}{s} \end{bmatrix} \Rightarrow P(z) = \begin{bmatrix} \frac{0.10517z^{-5}}{z-1.105} & \frac{0.04758z^{-7}}{z-0.9048} \\ \frac{0.000995z^{-3}}{z-0.99} & \frac{0.1}{z-1} \end{bmatrix}$$

$$G^\dagger(z) = \begin{bmatrix} \frac{0.063786}{z-1.105} & \frac{0.095829}{z-0.9048} \\ \frac{0.0010249}{z-0.99} & \frac{0.1}{z-1} \end{bmatrix}$$

Design example

5. MIMO plants

.

For $G^\dagger(z)$,

- Design the controller: $K(z) = \begin{bmatrix} \frac{3.8625(z-0.9823)}{z-1} & 0.97215 \\ -0.070359 & 0.58583 \end{bmatrix}$

  leading to $H(z) = P(z)K(z)[I + G^\dagger(z)K(z)]^{-1}$

  with static gain: $H(1) = \begin{bmatrix} 1.6487 & 0 \\ 0.0050 & 1.0002 \end{bmatrix}$

- Choose the IMC controller such that $Q = Q_0$ with

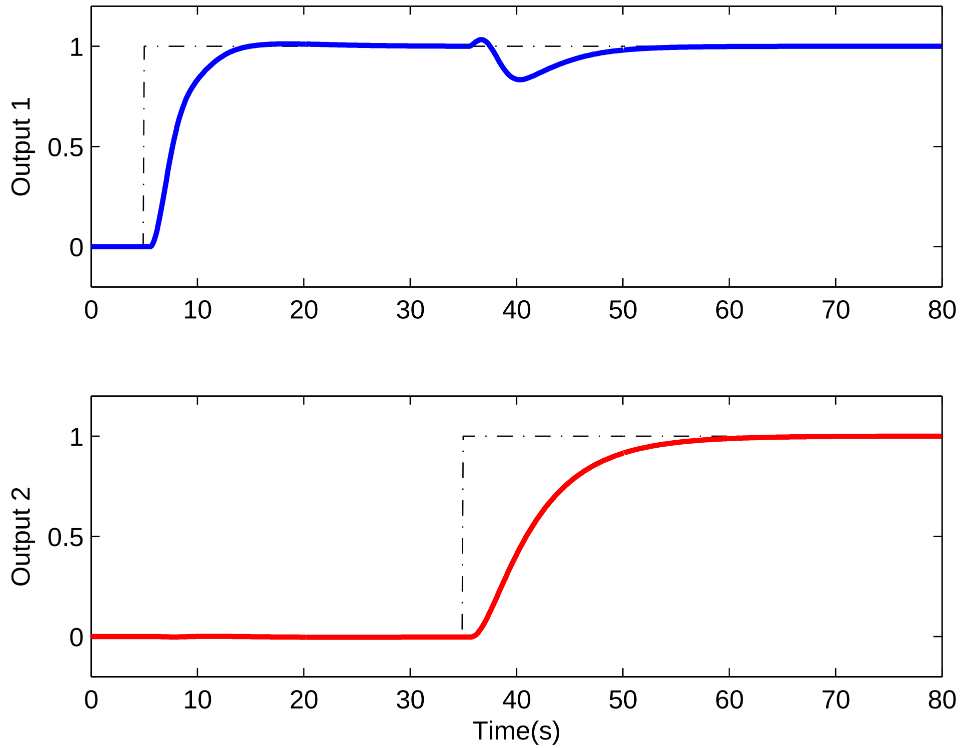
$$Q_0 = H(1)^{-1} = \begin{bmatrix} 0.6065 & 0 \\ -0.0030 & 0.998 \end{bmatrix}$$

  and prefilter: $F(z) = \begin{bmatrix} \frac{0.0198}{z-0.9802} & 0 \\ 0 & \frac{0.0198}{z-0.9802} \end{bmatrix}$

Design example

5. MIMO plant

- Preliminary results



Step responses. K(z) being designed by loop shaping.

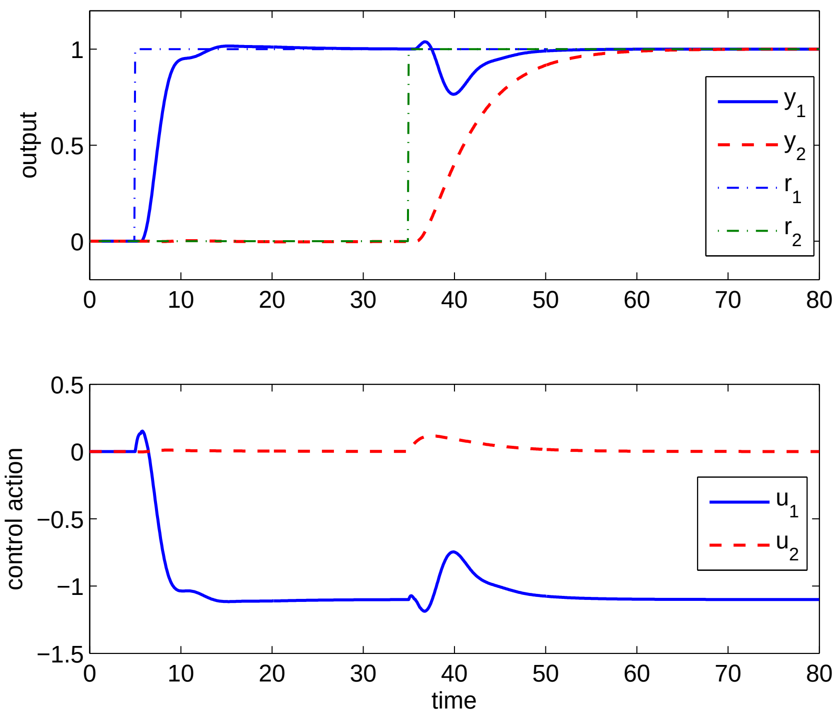
Design example

5. MIMO plants

- Improved robustness. For $P_p(s) = \begin{bmatrix} \frac{e^{-0.6s}}{s-1.1} & \frac{0.6e^{-0.8s}}{s+1.1} \\ \frac{0.1e^{-0.3s}}{10s+1} & \frac{1.1e^{-0.7s}}{s} \end{bmatrix}$

  The system becomes unstable with previous K and Q.

- Design $Q = Q_0 F_0$ with $F_0(z) = \begin{bmatrix} \frac{0.00995}{z-0.99} & 0 \\ 0 & \frac{0.00995}{z-0.99} \end{bmatrix}$



Decoupling

5. MIMO plants

$P(z) = [g_{ij}(z) z^{-d_{ij}}]$

- Dynamic precompensator: $P(z) = M(z) K_d^{-1}(z)$, $M(z)$ being diagonal. All matrices making $M = K_d P$ diagonal are $K_d = adj(P)K$, for any K.
- State Feedback (no delay): For $x_{k+1} = Ax_k + Bu_k; \quad y_k = Cx_k$

$$\begin{aligned} u_k &= K_D x_k + F v_k; \quad F = \begin{bmatrix} {}_1CA^{r_1-1}B \\ {}_2CA^{r_2-1}B \\ \vdots \\ {}_mCA^{r_m-1}B \end{bmatrix}^{-1} \\ K_D &= -F \begin{bmatrix} {}_1CA^{r_1} \\ {}_2CA^{r_2} \\ \vdots \\ {}_mCA^{r_m} \end{bmatrix} \Rightarrow y(z) = \begin{bmatrix} z^{-r_1} & 0 & 0 & 0 \\ 0 & z^{-r_2} & 0 & 0 \\ \vdots & & & \vdots \\ 0 & 0 & 0 & z^{-r_m} \end{bmatrix} v(z) \end{aligned}$$

Decoupling

5. MIMO plants

- Approximated decoupling
  - Approximate a first+order plus time-delay of the system (each TF)
  - Approximate any time constant in each column $(1+\tau_{ij}s)$ by $(1+\tau_{sj}s((1+\tau_{ij}-\tau_{sj})s$ where $\tau_{sj} = \min_i \tau_{ij}$
  - Start with $K = I$, then $K_d = adj(P)$
  - Extract the larger common time delay and time constant from any column of K_d.
  - Apply a controller design for each diagonal element

Example $P(s) = \begin{bmatrix} \frac{3}{1+9s}e^{-3s} & \frac{2}{1+6s}e^{-2s} \\ \frac{1}{1+5s}e^{-4s} & \frac{2}{1+7s}e^{-4s} \end{bmatrix} \Rightarrow K_d(s) = \begin{bmatrix} \frac{2}{1+2s} & -2 \\ -1 & \frac{3}{1+3s}e^{-s} \end{bmatrix}$

Leading to

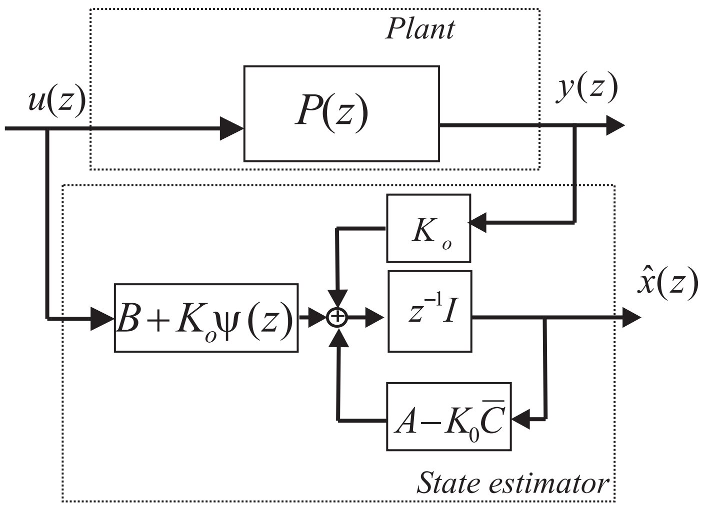
$$M(s) = \begin{bmatrix} -\frac{2}{1+6s}e^{-2s} + \frac{6}{(1+6s)(1+3s)(1+2s)}e^{-3s} & 0 \\ 0 & \frac{-2}{1+5s}e^{-4s} + \frac{6}{(1+5s)(1+3s)(1+2s)}e^{-5s} \end{bmatrix}$$

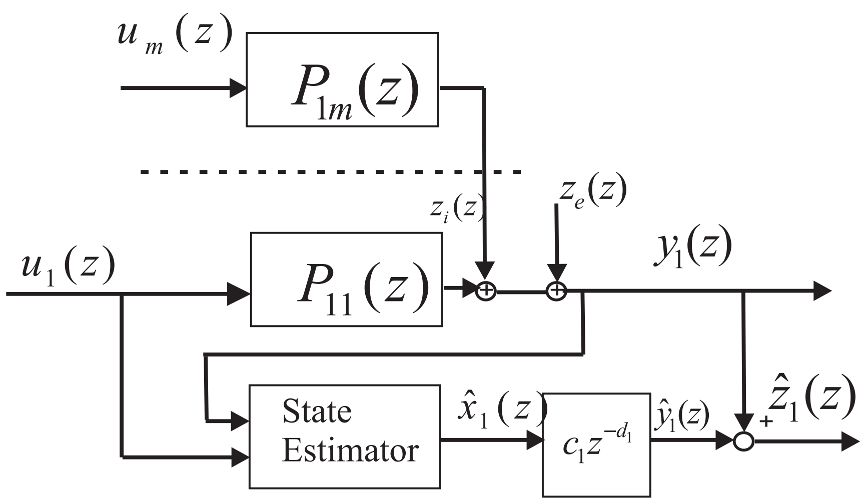
High cost for decoupling!!

Decoupling

5. MIMO plants

- Disturbance cancelation
  - Estimate the state
  - Estimate the “single" outputs and observe the “disturbances":

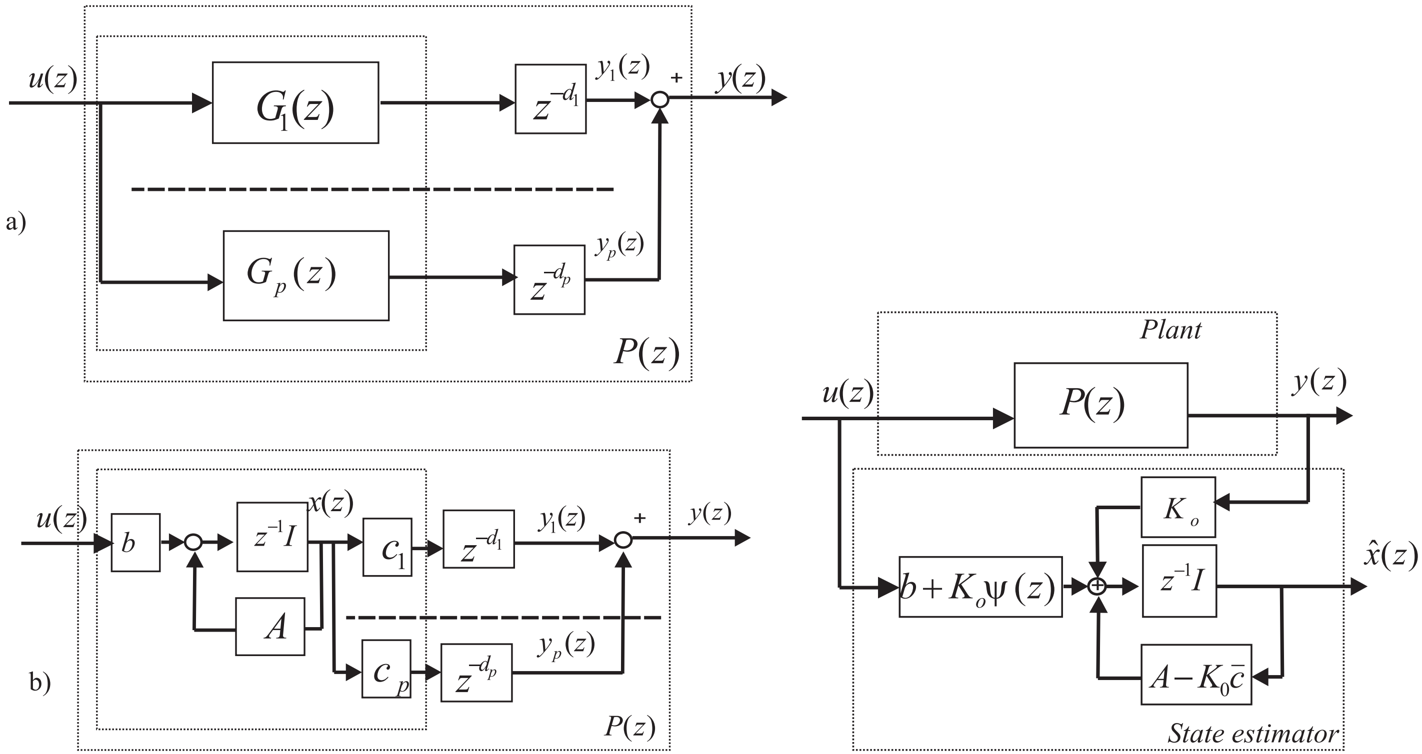




Multidelays

5. MIMO plants

- Optimal control: $y(z) = \sum_{i=1}^{p} c_i [zI - A]^{-1} b z^{-d_i} u(z)$



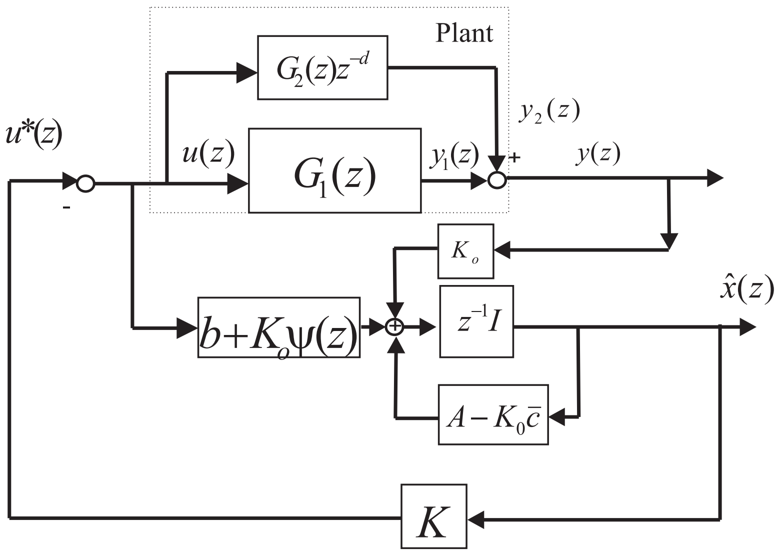
Multidelays: Example

5. MIMO plants

- Assume: $y(s) = \frac{5+(1+s)e^{-0.6s}}{(1+2s)(1+1.5s)}$, $h = 0.1s$

$$G_1(z) = \frac{0.0080163(z+0.9619)}{(z-0.9512)(z-0.9355)} = \frac{0.008016z+0.007711}{z^2-1.887z+0.8899}$$

$$G_2(z) = \frac{0.033048(z-0.9048)z^{-6}}{(z-0.9512)(z-0.9355)} = \frac{0.03305z-0.0299}{z^2-1.887z+0.8899}z^{-6}$$



$$A = \begin{bmatrix} 1.8867 & -0.8899 \\ 1 & 0 \end{bmatrix}; \quad b = \begin{bmatrix} 0.125 \\ 0 \end{bmatrix};$$

$$c_1 = \begin{bmatrix} 0.0641 & 0.0617 \end{bmatrix}; c_2 = \begin{bmatrix} 0.1322 & -0.1196 \end{bmatrix}$$

$$\bar{c} = c_1 + c_2 A^{-6}$$

Multidelays: Example

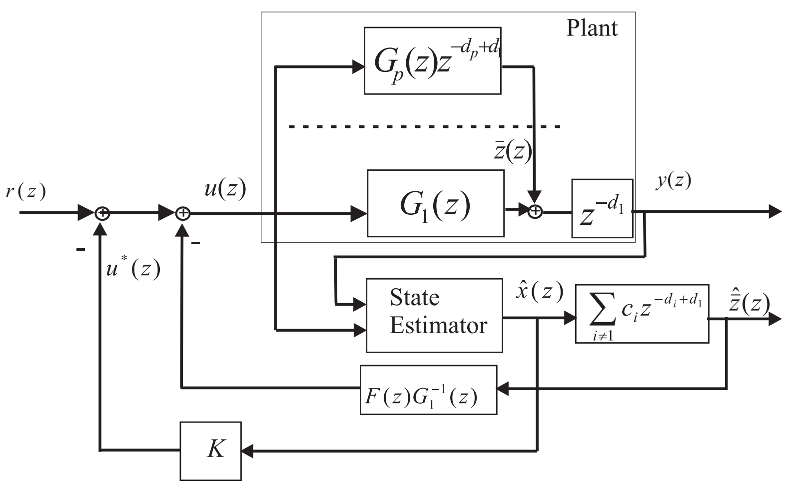
5. MIMO plants

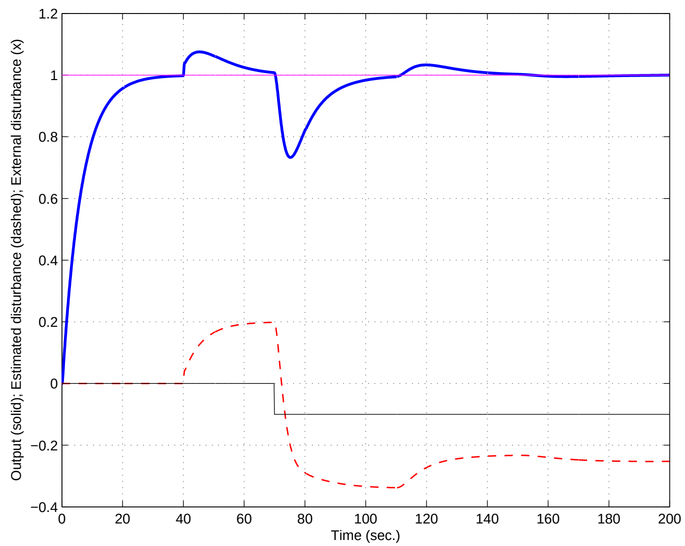
- Disturbance rejection $y(z) = \sum_{i=1}^{p} c_i[zI - A]^{-1} b z^{-d_i} u(z)$

  $G_1(z) = \frac{0.0080163(z+0.9619)}{(z-0.9512)(z-0.9355)}; G_1^{-1}(z) = \frac{(z-0.9512)(z-0.9355)}{0.0080163(z+0.9619)}$

  $G_2(z) = \frac{0.033048(z-0.9048)z^{-400}}{((z-0.9512)(z-0.9355)}; \quad L = 40s;$ Input disturbance at $t = 70s$

  $F(z) = \frac{(1-\lambda)}{(z-\lambda)}; \quad \lambda = 0.01$





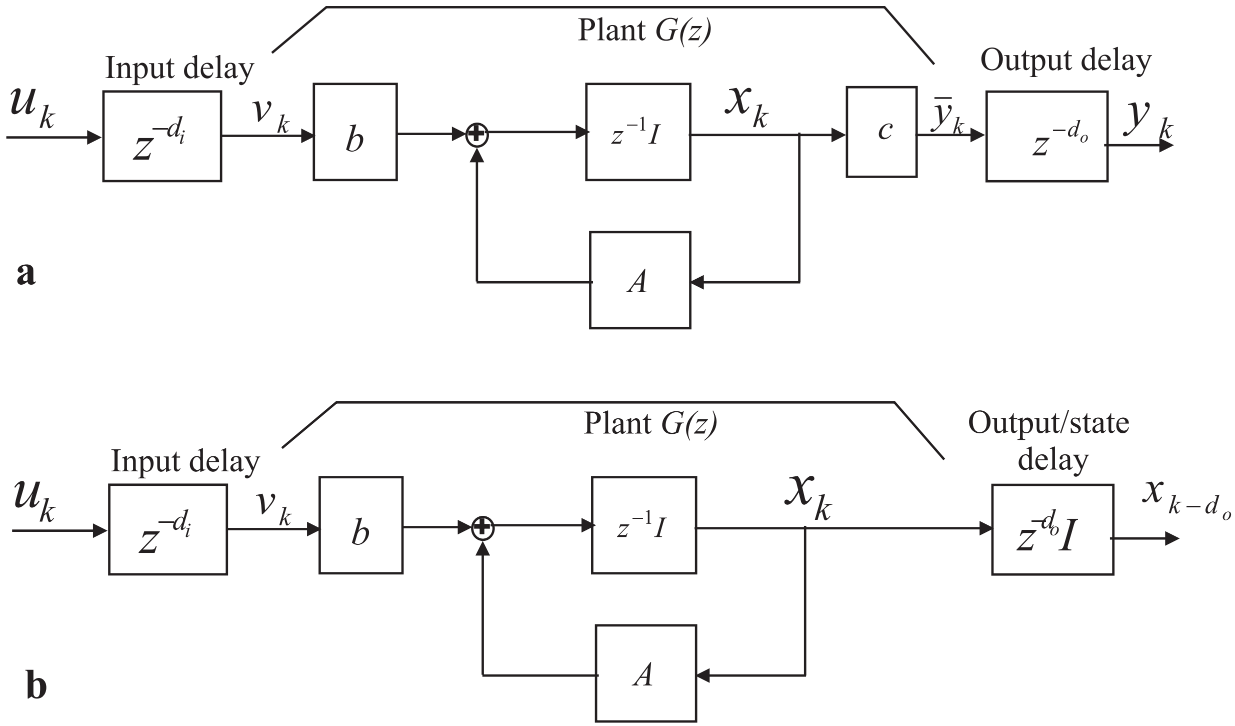
6. Some applications

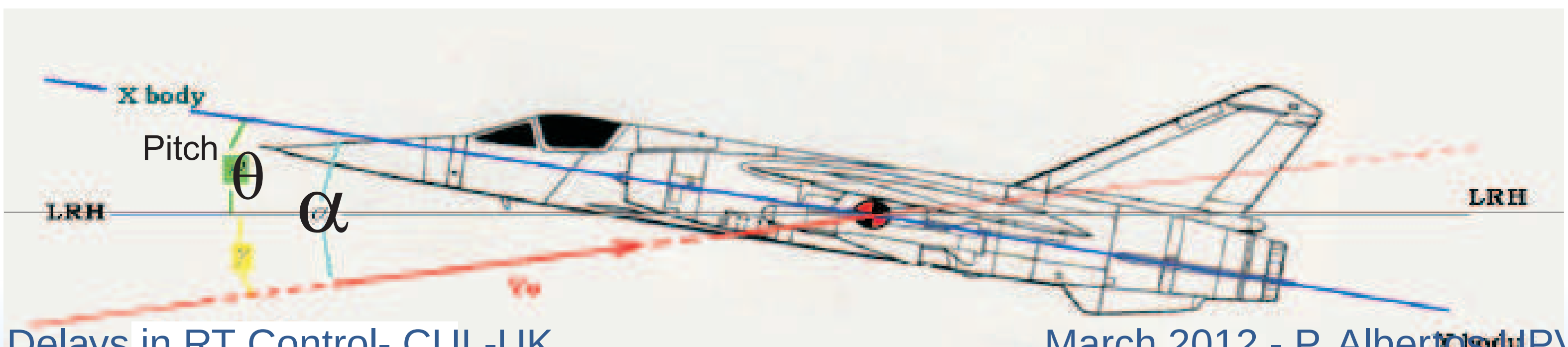
1. Introduction and motivation
2. Models and control issues
3. Classical time delayed plants control
4. Smith predictor improvements
5. MIMO Plants.
6. Some applications
   - Aircraft Pitch control
   - CSTR control
   - Steel rolling mill
   - Networked control systems
7. Conclusions and **open issues**

Pitch Angle Control

6. Some Applications

- Assume a simplified longitudinal model of the aircraft





Pitch Angle Control

6. Some Applications

.

State variables: the angle of attack α, the pitch rate q and the pitch angle θ, and the control variable is the elevator deflection δ_e.

$$A = \begin{bmatrix} 0.95122 & 0.042956 & 0 \\ 0.039026 & 0.94844 & 0 \\ 0.056289 & 2.7608 & 1 \end{bmatrix}; b = \begin{bmatrix} -0.00029667 \\ -0.00855 \\ -0.012226 \end{bmatrix}$$

Control index

$$J = \sum_{k=0}^{\infty} [\alpha_k^2 + 10q_k^2 + \theta_k^2 + 0.1\delta_{e,k}^2]; \Rightarrow u_k^* = Kx_k$$

Measurement delay ($d_o = 2$): $x_k = A^{d_o} x_{k-d_o} + W_{d_o} U_{k-d_o}$

where

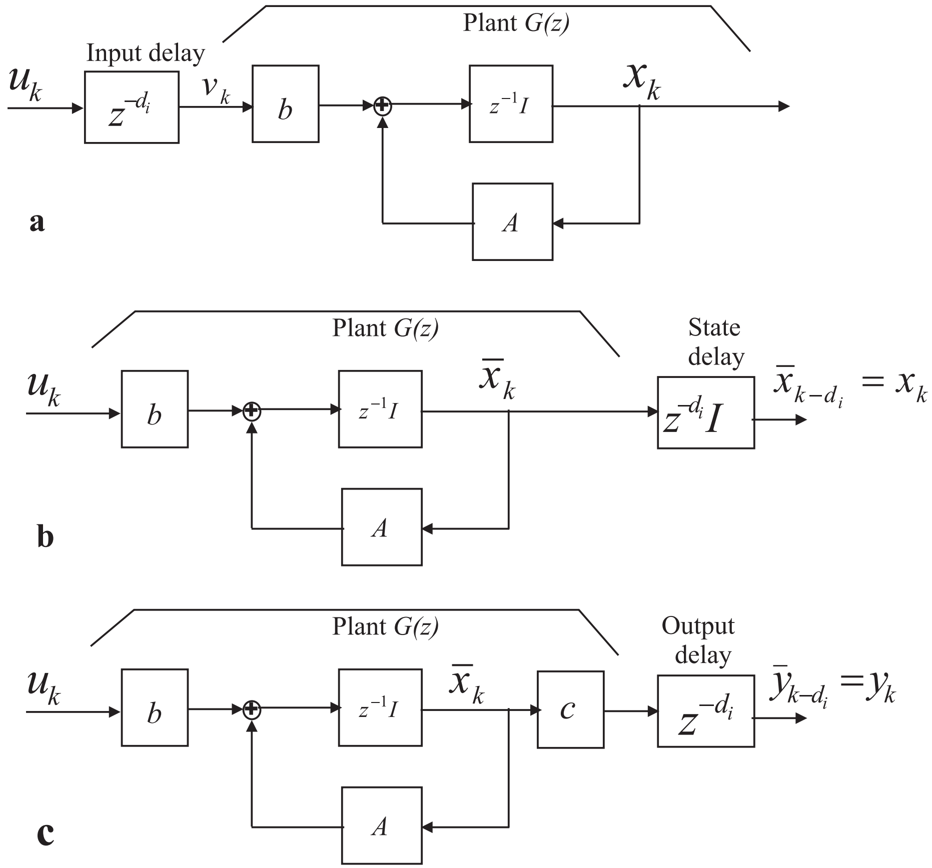
$$W_{d_o} = W_2 = \begin{bmatrix} -0.0003 & -0.0006 \\ -0.0086 & -0.0081 \\ -0.0122 & -0.0358 \end{bmatrix}; \quad A^{d_o} = A^2 = \begin{bmatrix} 0.9065 & 0.0816 & 0 \\ 0.0741 & .9012 & 0 \\ 0.2176 & 5.3817 & 1 \end{bmatrix}$$

$$u_k^* = Kx_k = \begin{bmatrix} 4.4216 & 37.2849 & 2.6412 \end{bmatrix} x_k \quad \Rightarrow u_k^* = K[A^{d_o} x_{k-d_o} + W_{d_o} U_{k-d_o}]$$

Pitch Angle Control

6. Some Applications

- Input delay: $d_i = 4$



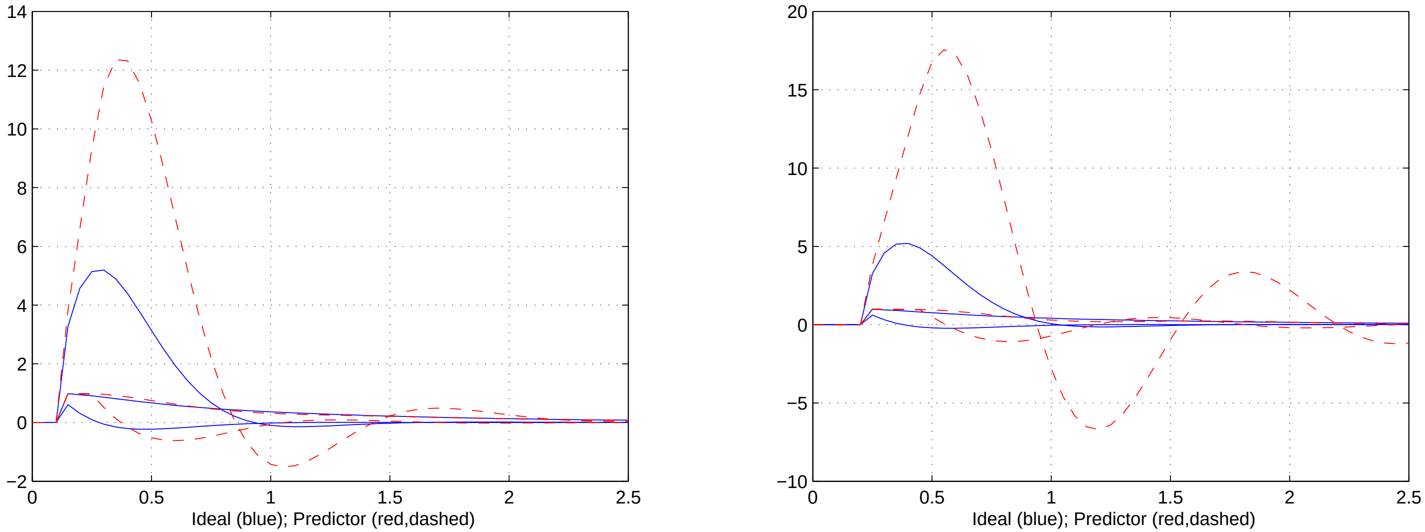
$$u_k^* = K[W_4 U_{k-4} + A^4 x_k]; \quad W_4 = \begin{bmatrix} -0.0003 & -0.0006 & -0.0010 & -0.0013 \\ -0.0086 & -0.0081 & -0.0077 & -0.0074 \\ -0.0122 & -0.0358 & -0.0583 & -0.0797 \end{bmatrix}$$

Pitch Angle Control

6. Some Applications

- Results

  a) Output delay: $d_o = 2;$ b) Input delay: $d_i = 4$



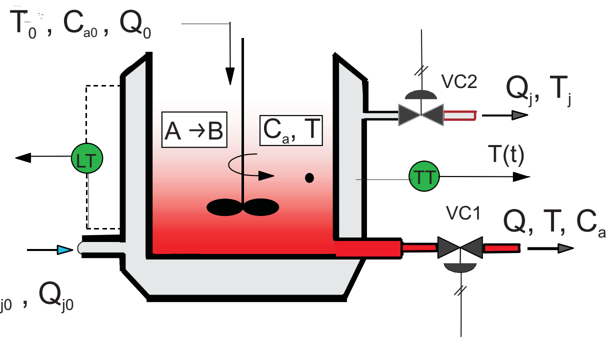
a) b)

OPTIMAL CONTROL APPLICATIONS AND METHODS Optim. Control Appl. Meth. (2011) Published online in Wiley Online Library (wileyonlinelibrary.com). DOI: 10.1002/oca.1007

CSTR Control

6. Some Applications

- Assume a simplified linear reduced order DT model of the CSTR



$$y_k = \begin{bmatrix} x_{1,k-d_{o,1}} \\ x_{2,k-d_{o,2}} \end{bmatrix}$$

$$x_{k+1} = Ax_k + A_d x_{k-d} + B \begin{bmatrix} u_{1,k-d_{i,1}} \\ u_{2,k-d_{i,2}} \end{bmatrix}$$

- with the following matrices: $B = \begin{bmatrix} 0.3776 & 0.08428 \\ -3.878 & -6.6 \end{bmatrix}$

$$A = \begin{bmatrix} 0.7442 & -0.03663 \\ 4.576 & 1.735 \end{bmatrix}; \quad A_d = \begin{bmatrix} 0.01819 & -0.000355 \\ 0.04433 & 0.02779 \end{bmatrix};$$

CSTR Control

6. Some Applications

- Output delays are treated as before
- Input delays, a state vector is associated with each input

$$\begin{aligned}
\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}_{k+1} &= \begin{bmatrix} A & 0 \\ 0 & A \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}_k + \begin{bmatrix} B_1 & 0 \\ 0 & B_2 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}_k \\
&= \bar{A} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}_k + \bar{B} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}_k
\end{aligned}$$

  and the process state is given by $x_k = x_{1,k-d_{i,1}} + x_{2,k-d_{i,2}}$
  Each delayed partial state can be computed by applying single delay for each input
- State delay is compensated by:

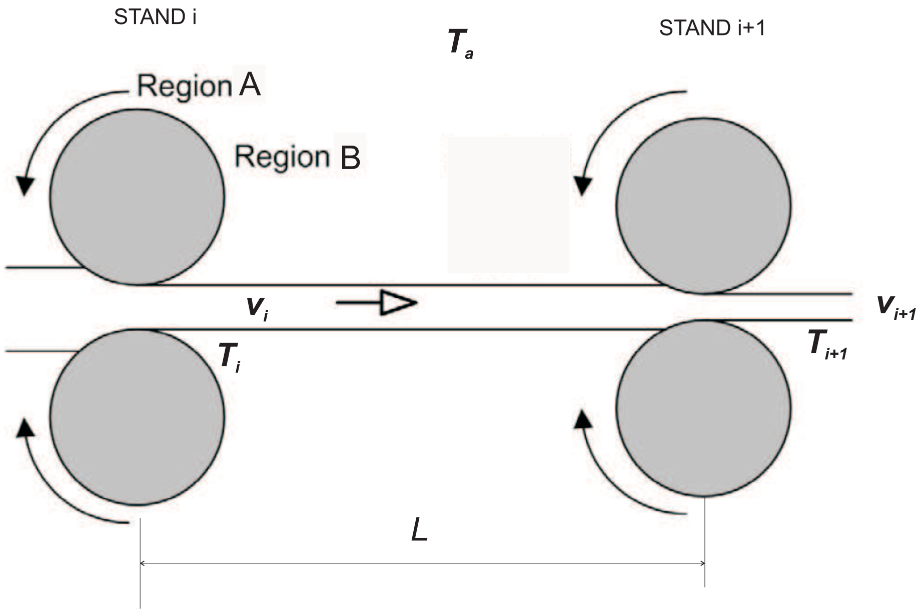
$$u_k = Kx_k + K_d x_{k-d}$$

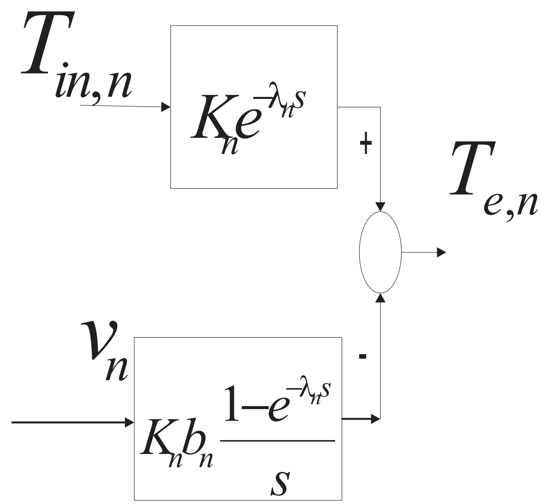
  allowing to assign the poles of the controlled plant provided that
  - *Controllability*. The pair (A, B) is controllable
  - *Delay cancelation*. There exists K_d such that $A_d + BK_d = 0$.

Hot Rolling Mill

6. Some Applications

- Temperature model





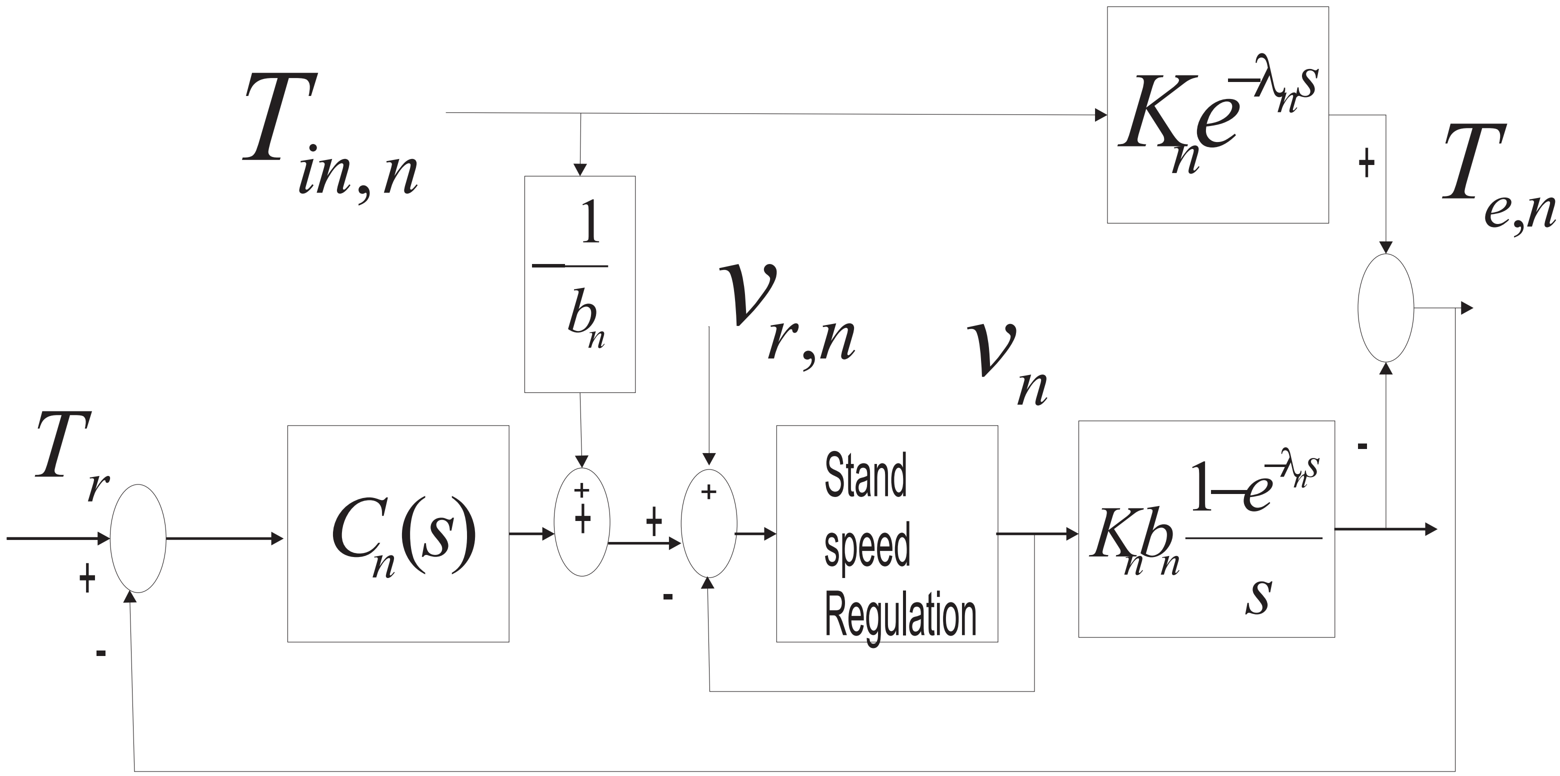
$$\frac{\partial T_i(t,x)}{\partial t} = -v_i \frac{\partial T_i(t,x)}{\partial x} + a[T_a - T_i(t,x)] + b[T_a^4 - T_i(t,x)^4]; \quad T_i(t,0) = T_{in,i}(t)$$

$$G_T^i = K_i e^{-\lambda_i s}; \quad G_v^i = -\frac{b_i K_i}{s}(1 - e^{-\lambda_i s})$$

Hot Rolling Mill

6. Some Applications

- Control schema: single stand

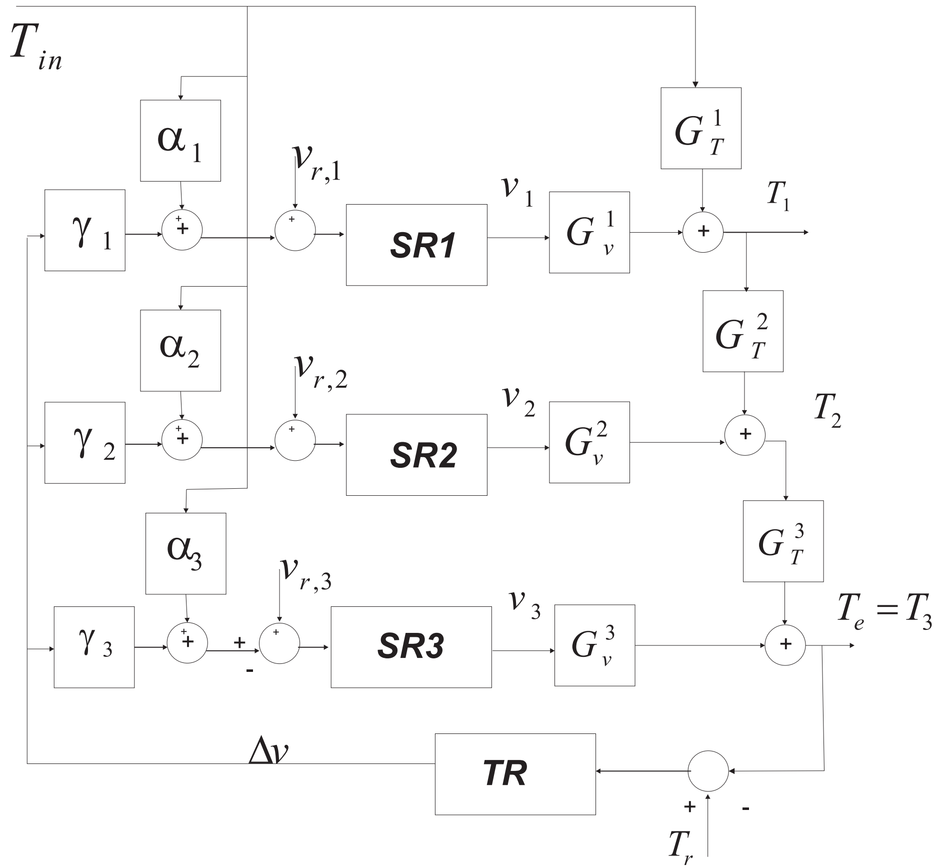


- $C_n(s)$ is a PI controller (sliding mode, IMC...) to counteract changes in exit temperature

Hot Rolling Mill

6. Some Applications

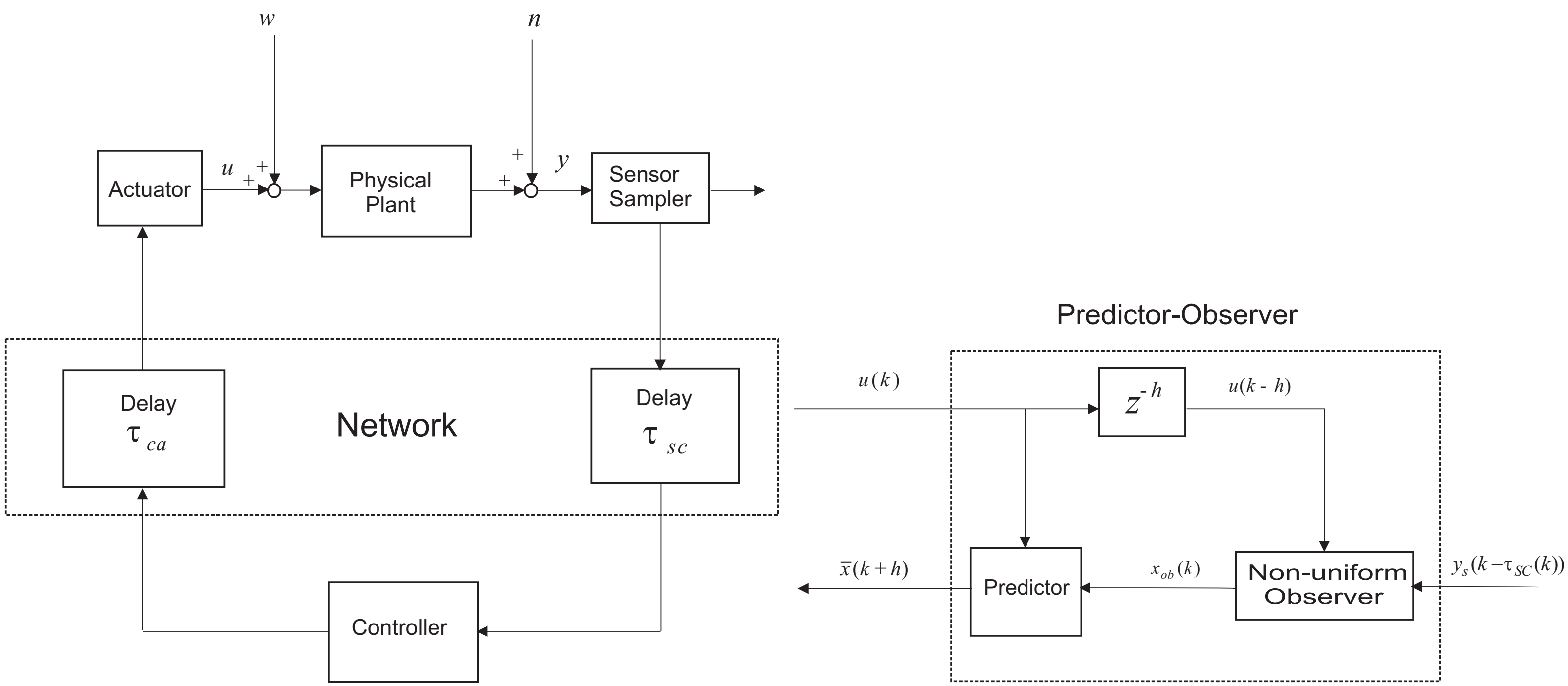
- Control schema: 3-stand



- α weights the stand temperature changes due to inlet temperature changes
- γ apportions the speed changes based on the pre-scheduled reduction.

Networked Control Systems

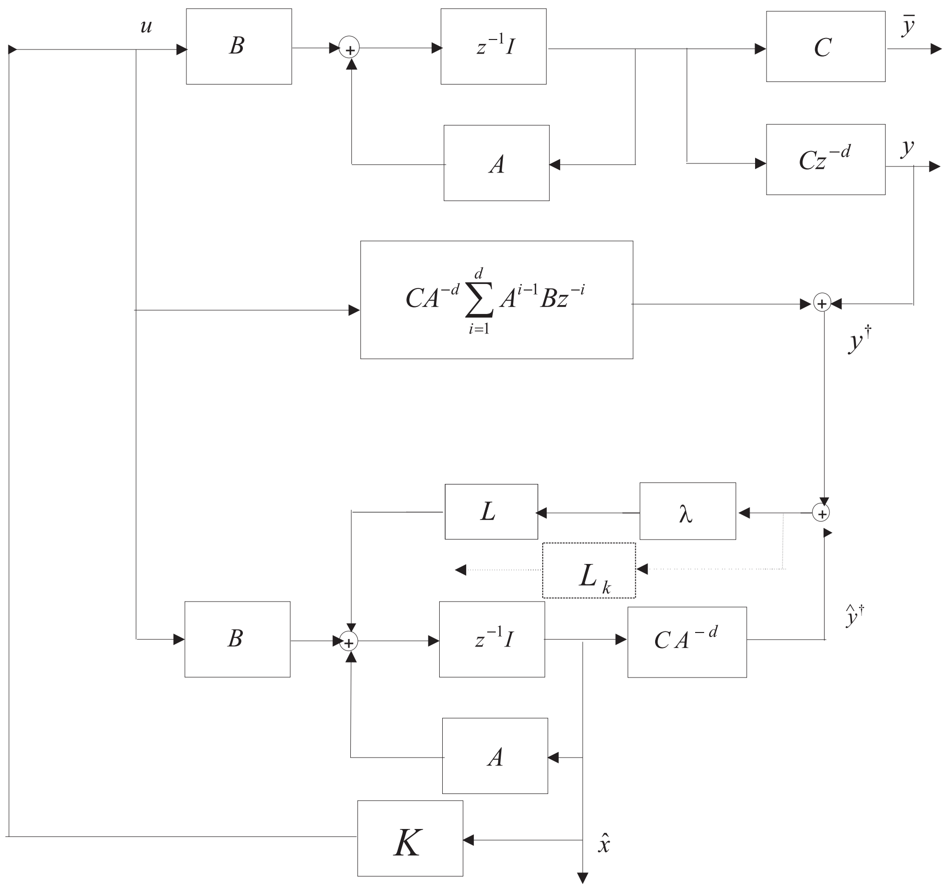
6. Some Applications



- NCS: Delay is random
- Data missing. Packet drop
- Undelayed signal estimator

Networked Control Systems

6. Some Applications



- Data missing.
- Fixed delay

7. Conclusions

Conclusions

- Time delays (as non linearities) are unavoidable and (usually) degrade performance
- There is not a unique treatment for time delays (as for non linear systems)
- Single input/output delays are easily treated, even for unstable (NMP) plants
- State (and distributed) delays require deeper analysis
- Additional delays, as well as cross-loops delays can be counteracted
  - By feedforward compensation based on estimations
  - This requires very good plant models
- Stabilizing and optimal control are feasible
- Tracking and disturbance rejection are questionable
- Random and time varying delays (NCS) require stochastic tools

Time Delays in Real-Time control

An overview

Pedro Albertos

pedro@aii.upv.es

Department of Systems Engineering and Control

Instituto Universitario de Automática e Informática Industrial

Universitat Politècnica de València